COURSES

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

Call for Resources

JUNE 1996

MATHEMATICS 12

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The Common Curriculum Framework

for

K–12 MATHEMATICS

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MATHEMATICS 12:
GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY STRAND AND SUBSTRAND

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by strand and substrand, for the Mathematics 12 course.

The coding for mathematical processes follows the same scheme as in the *Common Curriculum Framework*.

CLUSTERS IN THE MATHEMATICS 12 COURSE

There are 5 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

The common cluster, numbered C6, is part of the mathematics expected of all students completing a K to 12 mathematics program.

Pure clusters, numbered P6 to P9, place more emphasis on precise mathematical theory. The approaches used are primarily algebraic and graphical.

CODING FOR ILLUSTRATIVE EXAMPLES (IEs)

The illustrative examples (IEs) listed on the following pages are organized by strand and substrand and have been correlated to specific outcomes (SOs). The numbers are taken directly from the *Common Curriculum Framework*.

NUMBERING SYSTEM

The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59 of the *Common Curriculum Framework*). For example, C2 – 6. (PR53) is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

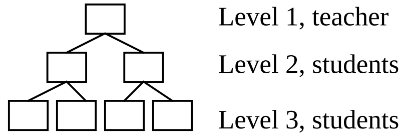
General Outcomes	Specific Outcomes	Illustrative Examples
Generate and analyze exponential patterns. *(continued)*	P6–1. (PR19) Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems. [CN, R, T]	1.1 Determine the n^{th} term and the sum of the first n terms of the geometric sequence whose first three terms are 2, 6 and 18. 1.2 Mathematicians use sigma notation as a way to write the sum of a series. For example: $\sum_{k=1}^{5} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$. Use sigma notation to write the series 5 – 15 + 45 – . . . + 3645. 1.3 Suppose that a principal of P dollars is invested at an annual interest rate r that is compounded annually. The amount A after t years is given by $A = P(1 + r)^t$. a) Find the number of years for the amount to double, if $2000 is invested at a rate of 7.5%, compounded annually. b) If the interest rate were 7.25% per annum, compounded semi-annually, how would the doubling period change? c) What would be the doubling period, if the rate were 7% per annum, compounded daily? 1.4 For the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$, find the sum of 20 terms. 1.5 The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years. a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates: • 4% per annum, compounded annually • 8% per annum, compounded annually • 24% per annum, compounded annually. b) What general conclusion can be drawn as to the accuracy of rule of 72 calculations?

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P6–2. (PR20) Connect geometric sequences to exponential functions over the natural numbers. [E, R, V]	2.1 The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. a) Calculate the population in 1998, 2009, 2019. b) Calculate the population that food production could sustain in 1998, 2009, 2019. c) When will the population exceed the food supply? 2.2 The following is a school trip telephoning tree.  a) At what level are 64 students contacted? b) How many are contacted at the 8th level? c) By the 8th level how many students, in total, have been contacted? d) By the *n*th level how many students, in total, have been contacted? e) If there are 300 students in total, by what level will all have been contacted?
	P6–3. (PR21) Estimate values of expressions for infinite geometric processes. [PS, R, T]	3.1 For the infinite series $2+\frac{2}{5}+\frac{2}{25}+\ldots$, estimate the sum to four decimal places. 3.2 An oil well produces 25 000 barrels of oil during its first month of production. If its production drops by 5% each month, estimate the total production before the well runs dry.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve exponential, logarithmic and trigonometric equations and identities.	P6–4. (PR39) Solve exponential equations having bases that are powers of one another. [E, R]	4.1 Solve for *x*: $3^{(4x-1)} = 27^{2x}$. 4.2 A string of ones and zeros is the binary representation of a number. If this number is converted to the base-16 hexadecimal representation, it is 9 digits shorter. As well, the decimal and hexadecimal representations have the same number of digits. a) How many digits are there in the binary representation of the original number? b) Between what two decimal numbers does the original number lie?
	P6–5. (PR40) Solve and verify exponential and logarithmic equations and identities. [R]	5.1 Solve for *x*: $\log_2(x-2) + \log_2(x) = \log_2(3)$. 5.2 Solve for *x*: $2 \times 3^x = 5^{(x-1)}$. 5.3 Solve for *x*, checking for any extraneous solutions: $\log_5(3x+1) + \log_5(x-3) = 3$. 5.4 The pH of an acid is given by $\text{pH} = -\log_{10}[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per litre. What is the hydrogen ion concentration of a weak vinegar solution of pH = 3.1? 5.5 Joe has $50 000 invested at an interest rate of 7% per annum, compounded monthly. Laura has $40 000 invested at 9.5% per annum, compounded annually. After how many years will the two investments be equal in value? 5.6 Verify the identity $\log_a\left(\frac{1}{x}\right) = -\log_a x$ for any base *a* and any positive value of *x*.

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
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[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Solve exponential, logarithmic and trigonometric equations and identities. *(continued)*	P8–1. (PR41) Distinguish between degree and radian measure, and solve problems, using both. [CN, E]	1.1 Draw an angle of one radian, and show how its radius and arc length are related. 1.2 Convert the following angles to degrees: $\frac{2\pi}{3}$, 1.6 rad. 1.3 Convert the following angles to radians expressed in terms of π: 180°, 55°. 1.4 In an experiment to verify the law of refraction, measurements were made of the angles of incidence and refraction. A spreadsheet was used to calculate the sines of both angles and the ratio of the two sines. The results of the spreadsheet calculation are shown in the following table.

Angle of incidence *i* (degrees)	Angle of refraction *r* (degrees)	sin *i*	sin *r*	(sin *i*)/(sin *r*)
10	7	–0.544	0.657	–0.83
20	13	0.913	0.420	2.17
30	19	–0.988	0.150	–6.59
40	25	0.745	–0.132	–5.63
50	30	–0.262	–0.988	0.27
60	35	–0.305	–0.428	0.71
70	38	0.774	0.296	2.61
80	40	–0.994	0.745	–1.33

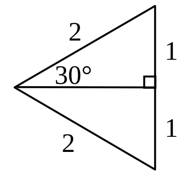
a) In the calculations of sin *i* and sin *r*, are the angles taken as being in degrees or in radians?
b) Modify the spreadsheet so that all entries reflect radian measure.
c) Modify the spreadsheet so that all entries reflect degree measure.
d) What conclusion can be drawn from either b) or c)?

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P8–2. (PR42) Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$. [CN, E]	2.1 Given an equilateral triangle with a side of 2 units, determine the exact trigonometric ratios of 30°.  2.2 Find the exact values for $\sin\frac{7\pi}{6}$, $\tan\frac{2\pi}{3}$, $\cos\frac{7\pi}{4}$.
	P8–3. (PR43) Solve first and second degree trigonometric equations over a domain of length 2π: • algebraically • graphically. [PS, T]	3.1 Find, algebraically and graphically, the solution to the following trigonometric equations: a) $1 + 2\cos x = 5\cos x$; $0 \le x < 2\pi$. Give solutions in decimal form. b) $\sin^2 x - \sin x = 0$; $0 \le x < 2\pi$. Give solutions as exact values. c) $\cos 4x = 0.5$; $0 \le x < 2\pi$. Give solutions as exact values.

Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P8–4. (PR44) Determine the general solutions to trigonometric equations where the domain is the set of real numbers. [PS, T]	4.1 Sketch the graph of $y = \sin 3x$. Use the graph to find all solutions of $\sin 3x = 0$ in the interval $0 \leq x < 2\pi$. 4.2 Use technology to graph $y = x - 2 \sin x$, and use the graph to find all solutions to the equation $2 \sin x = x$. Express answers to a three-decimal place accuracy. 4.3 What is the relation between the graphs of $y = \sin x$ and $y = \frac{1}{2}$ and the roots of the equation $0 = 2 \sin x - 1$? 4.4 Use technology to solve $\sin 3x = \frac{1}{2}$, and then write the general solution.
	P8–5. (PR45) Verify trigonometric identities: • numerically for any particular case • algebraically for general cases • graphically. [PS, R, T, V]	5.1 a) Verify that $\sin^2 x + \cos^2 x = 1$ for any real number x. b) Use this identity to show that $1 + \tan^2 x = \sec^2 x$ for any real number x, where $\cos x \neq 0$. 5.2 Given the identity $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$: a) verify the identity for the particular case when $x = \frac{\pi}{3}$ b) verify for a general angle, using an algebraic approach c) verify, by graphing the left-hand side and the right-hand side of the given identity.
	P8–6. (PR46) Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions. [R, T]	6.1 Write 2 (sin 5)(cos 5) in terms of a single trigonometric function. 6.2 Graph the function $f(x) = \frac{2 \tan x}{1 + \tan^2 x}$. a) Make a conjecture for the period of the above graph. b) Simplify the expression for $f(x)$ to a single trigonometric function, and then find the period of $f(x)$. c) Compare the solutions to a) and b).

Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze exponential and logarithmic functions, using technology as appropriate.	P6–6. (PR64) Graph and analyze an exponential function, using technology. [R, T, V]	6.1 Graph $y = 2^x$ with/without technology. 6.2 Graph $y = 4(2^x)$ and $y = 2^x$ on the same set of coordinate axes. Identify the domain, range, asymptotes and intercepts of each graph. What is the relationship between the two graphs?
	P6–7. (PR65) Model, graph and apply exponential functions to solve problems. [PS, T, V]	7.1 The summertime population of gophers in a field can be modelled by the equation $P = 100(1.1)^n$, where n is measured in years. Plot the graph for a 10-summer period, and use the graph to find out how long it takes for the gopher population to double. 7.2 The half-life of sodium-24 is 14.9 hours. Suppose that a hospital buys a 40 mg sample of sodium-24. a) How much of the sample will remain after 48 hours? b) How long will it be until only 1 mg remains? 7.3 The population of a certain country is 28 million and grows at a rate of 3% annually. Assuming the population is continuously growing, the population P, in millions, t years from now can be determined by the formula $P = 28e^{0.03t}$. a) In how many years will the population be 40 million? b) What factors could contribute to the breakdown of this model?
	P6–8. (PR66) Change functions from exponential form to logarithmic form and vice versa. [CN]	8.1 Rewrite $y = 2^x$ as a logarithmic function. 8.2 The ionization of pure water is shown in the equations: $[H^+]\,[OH^-] = 1.0 \times 10^{-14}$ and $[H^+] = [OH^-]$. If the pH of any solution is defined as $pH = -\log_{10}[H^+]$, what is the pH of pure water?
(continued)	P6–9. (PR67) Use logarithms to model practical problems. [CN, PS, V]	9.1 Research the strength of earthquakes, and compare them, using the Richter scale. 9.2 The Armenian earthquake, Richter scale 6.9, produced 3.5×10^{13} J of energy. How much energy did the Alaska earthquake, Richter scale 8.2, produce?

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P6–10. (PR68) Explain the relationship between the laws of logarithms and the laws of exponents. [C, T]	10.1 Explain how the exponent law $a^x \times a^y = a^{(x+y)}$ is related to the logarithmic law $\log_a(MN) = \log_a M + \log_a N$. 10.2 Use a calculator to find $\log_5 8$, and justify your procedure.
	P6–11. (PR69) Graph and analyze logarithmic functions with and without technology. [R, T, V]	11.1 Graph $y = \log_{10} x$ and $y = \log_2 x$ on the same set of coordinate axes. What is the likely position of the graph of $y = \log_5 x$? 11.2 Analyze the graph of $y = \log_{10}(2x + 3)$. Identify the domain, range, asymptotes and intercepts.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication
[CN] Connections
[E] Estimation and Mental Mathematics
[PS] Problem Solving
[R] Reasoning
[T] Technology
[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Represent and analyze trigonometric functions, using technology as appropriate.	P8–7. (PR70) Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position. [PS, R, V]	7.1 Triangle *OBA* has vertices *O*(0, 0), *B*(4, 0) and *A*(4, 3). The unit circle, centred at (0, 0), intersects *OA* at point *P*. a) Use similar triangles to find the coordinates of point *P*. b) Use trigonometric ratios to find the sine and cosine of angle *AOB*. c) Compare your results in b) to the coordinates of point *P*.
	P8–8. (PR71) Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for: • amplitude, if defined • period • domain and range • asymptotes, if any • behaviour under transformations. [CN, T, V]	8.1 Using a graphing utility, graph $y = \sin x$ and $y = \cos x$ on the same set of axes. a) What relationship seems to exist between the two? b) What is the amplitude and period of each graph? 8.2 Graph $y = \tan x$ and $y = \tan 2x$. Compare the period, the domain and the range of $y = \tan x$ to those of $y = \tan 2x$. 8.3 In the equation $y = A \sin [B(x + C)] + D$; $A = 4$, $B = 3$, $C = \frac{-3\pi}{4}$ and $D = -3$. Compare the graph of this function to the graph of $y = \sin x$ with respect to domain, range, amplitude, period, *x* and *y* intercepts, horizontal phase shift and vertical displacement.
	P8–9. (PR72) Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for: • period • domain and range • asymptotes • behaviour under transformations. [CN, T, V]	9.1 Graph and analyze: a) $y = \sec x$ b) $y = \csc x$ c) $y = \cot x$. 9.2 Compare the domain, range and period of: a) $f(x) = \csc x$ and $g(x) = 5 \csc x$ b) $f(x) = \cot x$ and $g(x) = \cot 2x$.
(continued)		

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

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General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P8–10. (PR73) Use trigonometric functions to model and solve problems. [PS, R, V]	10.1 For a Saskatchewan town, the latest sunrise time is on December 21, at 09:15. The earliest sunrise time is on June 21, at 03:15. Sunrise times on other dates can be predicted from a sinusoidal equation. **Note**: There is no Daylight Saving Time in Saskatchewan. a) What is the equation that describes sunrise times? b) What is the amplitude and period of the equation describing sunrise times? c) Use the equation to predict the time of sunrise on April 9. d) What is the average time of sunrise throughout the year? 10.2 The depth of water in a harbour is given by the equation $d(t) = -4.5 \cos(0.16\pi t) + 13.7$, where $d(t)$ is the depth, in metres, and t is the time, in hours, after low tide. a) Sketch the graph of $d(t)$. b) What is the period of the tide, from one high tide to the next? c) A bulk carrier needs at least 14.5 m of water to dock safely. For how many hours per cycle can the bulk carrier dock safely? 10.3 The average daily maximum temperature in Vancouver follows a sinusoidal pattern with a highest value of 23.6°C on July 26, and a lowest value of 4.2°C on January 26. a) Describe this variation with a sine or cosine equation. b) What is the expected maximum temperature for May 26? c) How many days will have an expected maximum of 21.0°C or higher? d) Explain why different equations give the same answers for b) and c).

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Classify conic sections, using their shapes and equations.	P9–1. (SS35) Classify conic sections according to shape. [C, R, V]	1.1 Visualize the shapes generated from the intersection of a double-napped cone and a plane. For each conic section, describe the relationship between the plane, the central axis of the cone and the cone's generator.
	P9–2. (SS36) Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only). [CN, T, V]	2.1 A circle with a radius of 4 units has the equation $x^2 + y^2 - 16 = 0$. What are the values of A, C and F in the general form? What is the radius of the circle $25x^2 + 25y^2 - 100 = 0$? 2.2 a) Graph the circle $x^2 + y^2 = 25$. b) Graph $Ax^2 + y^2 = 25$ where $A > 1$. c) Graph $Ax^2 + y^2 = 25$ where $0 < A < 1$. d) Graph $Ax^2 + y^2 = 25$ where $A = 0$. e) Graph $x^2 + Cy^2 = 25$ where $C > 1$. f) Graph $x^2 + Cy^2 = 25$ where $0 < C < 1$. g) Graph $x^2 + Cy^2 = 25$ where $C = 0$. h) Draw a conclusion based on the results found in b) through g). 2.3 Graph $2x^2 + y^2 - 12 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph? 2.4 Graph $4x^2 - 25y^2 - 100 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph?
	P9–3. (SS37) Convert a given equation of a conic section from general to standard form and vice versa. [R, T]	3.1 Convert to standard form: a) $x^2 + y^2 + 6x - 8y = 11$ b) $3x^2 + y^2 + 6x + 4y = 9$. 3.2 Convert to general form: a) $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{16} = 1$ b) $\frac{(x+3)^2}{25} - \frac{(y-4)^2}{16} = 1$.

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

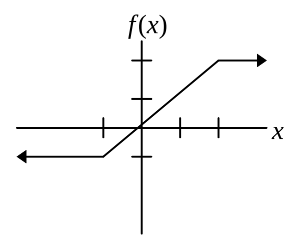
General Outcomes	Specific Outcomes	Illustrative Examples
Perform, analyze and create transformations of functions and relations that are described by equations or graphs.	P9–4. (SS38) Describe how various translations of functions affect graphs and their related equations: • $y = f(x - h)$ • $y - k = f(x)$. [C, T, V]	4.1 Describe how the graph of $y = x^2$ compares to the graph of $y = x^2 - 2$. 4.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $f(x) - 2$ b) $f(x - 2)$ c) $f(x - 2) + 1$.
	P9–5. (SS39) Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: • $y = af(x)$ • $y = f(kx)$. [C, T, V]	5.1 Describe how the graph of $y = x^2$ compares to the graph of: a) $y = 2x^2$ b) $y = \frac{2}{3}x^2$. 5.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $2f(x)$ b) $-2f(x)$ c) $\frac{2}{3}f(x)$. Discuss the changes. 5.3 Given the graph of $f(x) = \sin x$, sketch the graph of: a) $f(2x)$ b) $\frac{2}{3}f(x)$. 5.4 Given the graph of $f(x) = x^3$ and its image under the transformation $g(x) = 3f(x)$, find the equation describing $g(x)$.
(continued)		

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P9–6. (SS40) Describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations: • $y = f(-x)$ • $y = -f(x)$ • $y = f^{-1}(x)$. [C, T, V]	6.1 Graph any function $f(x)$. Sketch the graph of: a) $-f(x)$ b) $f(-x)$ c) $f^{-1}(x)$ d) $f^{-1}[f(x)]$. 6.2 If $g(x)$ is the reflection of $f(x)$ in the y-axis, write the equation of $g(x)$ in terms of $f(x)$.
	P9–7. (SS41) Using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$. [C, T, V]	7.1 Given $f(x) = 2x + 1$, sketch the graph of $f(x)$ and of $\frac{1}{f(x)}$. What happens to the x-intercepts of $f(x)$? 7.2 Sketch the graph of $f(x) = \sin x$, and sketch $\frac{1}{\sin x}$. 7.3 Sketch $\frac{1}{f(x)}$, if $f(x)$ is shown by the accompanying sketch. 

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C] Communication
[CN] Connections
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General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P9–8. (SS42) Using the graph and/or the equation of $f(x)$, describe and sketch $\|f(x)\|$. [C, T, V]	8.1 Given the graph of $f(x) = 2x + 1$, sketch $\|f(x)\|$. 8.2 Sketch $y = 3\|\sin x\|$. What is the period of this function? 8.3 Sketch $f(x) = \dfrac{1}{\|x^2 - 1\|}$. 8.4 An AC generator has a voltage given by $V = 170 \cos (120\pi t)$, where V is the voltage and t the time in seconds. A simple DC rectifier has voltage output given by $V = 170\|\cos(120\pi t)\|$. Sketch the output graphs for both devices, and describe the similarities and differences.
	P9–9. (SS43) Describe and perform single transformations and combinations of transformations on functions and relations. [C, T, V]	9.1 Given $f(x) = x^2$, sketch the graph of $f(x)$, and sketch the graph of $-2f(x - 1) + 3$. 9.2 Determine the equation of the ellipse $x^2 + 4y^2 - 25 = 0$, after each of the following transformations: a) translated two units to the right b) translated three units down c) expanded by a factor of two along the horizontal axis d) expanded by a factor of one quarter along the vertical axis. 9.3 Given the circle $x^2 + y^2 = 1$ and its image under a translation described by the ordered pair $(2, -3)$: a) write the equation of the image b) if a point $P(a, b)$ is on the graph of the circle $x^2 + y^2 = 1$ and $P'(a', b')$ is the transformed image of P, what are the coordinates of P' in terms of a and b?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[E]	Estimation and Mental Mathematics	**[T]**	Technology
		[V]	Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
Use normal and binomial probability distributions to solve problems involving uncertainty.	C6–1. (SP11) Find the population standard deviation of a data set or a probability distribution, using technology. [CN, E, T, V]	1.1 Measure the height of each student in a class, and calculate the mean and standard deviation. 1.2 A company uses an automated packaging device to produce 50-g bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn. 54, 50, 47, 50, 51, 50 53, 50, 47, 51, 50, 51 52, 49, 46, 52, 50, 49 52, 48, 48, 53, 49, 49 51, 48, 49, 52, 49, 50 a) Calculate the mean and standard deviation of this data. b) What problems will be encountered, if the standard deviation gets too high? Dottori et al., *Foundations of Mathematics 11*, p. 392. Adapted with permission.
	C6–2. (SP12) Use *z*-scores and *z*-score tables to solve problems. [PS, R, T, V]	2.1 The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL. a) Calculate the *z*-score for a can with a volume of 355 mL. b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL? c) What percentage of production will consist of cans having content volumes less than 355 mL? d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50 000?
(continued)	*(continued)*	

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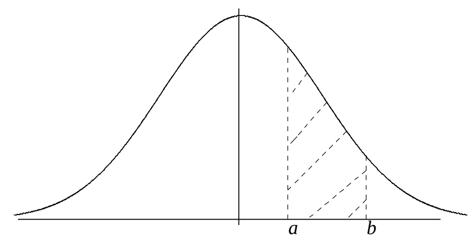
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	*(continued)*	2.2  a) What is the area under this curve? b) If $P(a < z < b) = 0.4$, what is the area under the curve for the interval $a < z < b$? c) If $P(z < b) = 0.9$, calculate $P(z > b)$, and calculate the value of b. 2.3 For entry into the Canadian Armed Forces, the standards for height used to be set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively. 2.4 A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find: a) the expected number of people with temperatures above 37.0°C b) the expected number of people with temperatures below 36.0°C. Also, estimate the range of temperatures contained within the sample. 2.5 In the general population, the IQ scores of individuals is normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested: a) What proportion of this group is expected to have IQs between 100 and 120? b) What is the probability that an individual in the group has an IQ greater than 120?

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General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	C6–3. (SP13) Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS]	3.1 The heights of males employed by a manufacturer follow a normal distribution with a mean of 169 cm and a standard deviation of 8 cm. a) Establish a symmetric 95% confidence interval for the average height in a random sample of 36 male employees. b) What happens to the width of the symmetric 95% confidence interval, if the sample size is increased from 36 to 225? 3.2 Pollsters estimate that the number of decided voters in favour of a particular bylaw is 64%, and the number opposed is 36%. a) If the sample size is 250, find the expected mean and standard deviation of *yes* voters. b) Estimate, for this sample, the expected percentage of *yes* voters, with a symmetric 95% confidence interval used to establish the margin of error. c) If the margin of error for the percentage of *yes* voters must be less than ±1.0%, what would be the minimum sample size required? 3.3 The probability that a car salesperson will complete a sale is 0.10. If the salesperson has 200 customers in the next month, establish a symmetric 95% confidence interval for the number of completed sales for the month.

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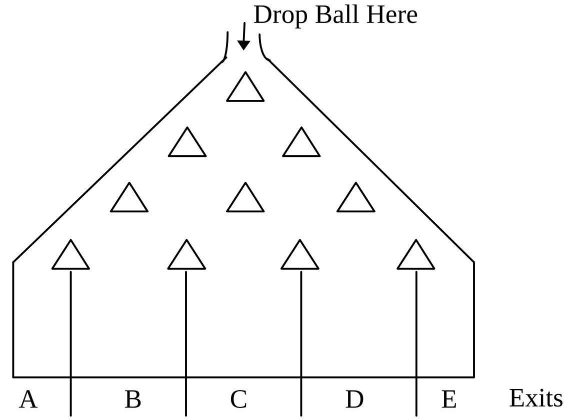
General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.	C6–4. (SP14) Solve pathway problems, interpreting and applying any constraints. [PS, R]	4.1 Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
	C6–5. (SP15) Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]	5.1 Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? 5.2 An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day had to be spent in Yellowknife?

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General Outcomes	Specific Outcomes	Illustrative Examples
Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.	P7–12. (SP16) Determine the number of permutations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]	12.1 List all possible permutations of the letters in the word **bold**. 12.2 Calculate the number of ways that an executive consisting of four people (president, vice-president, treasurer and secretary) can be selected from a group of 20 people. 12.3 Explain the meaning of ${}_8P_3$. Why does ${}_3P_8$ not make sense? 12.4 Develop and solve a problem where ${}_8P_3$ would be applicable. 12.5 Solve ${}_nP_2 = 30$. 12.6 On a 12-question multiple-choice test, three answers are A, three are B, three are C and three are D. How many different answer keys are possible?
	P7–13. (SP17) Determine the number of combinations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]	13.1 From a group of five student representatives, three will be chosen to work on the dance committee. a) List all possible committees. b) Calculate ${}_5C_3$, and compare to the answer in part a). c) If the committee had to have a chairperson, would it still be a combination? Why or why not? d) How many committees of three, with a chairperson, can be chosen from a group of 10 student representatives? 13.2 Show that ${}_nC_k = {}_nC_{(n-k)}$, using two different methods. Verify the truth of this assertion for the special case with $n = 10$ and $k = 3$. 13.3 How many diagonals are there in a regular polygon with 20 sides? What is the general formula for the number of diagonals in an n-sided polygon?
(continued)		

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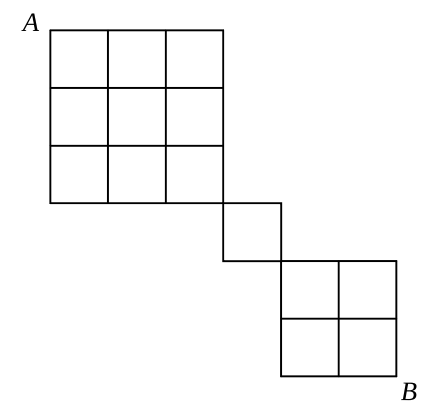
General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P7–14. (SP18) Determine the number of pathways in a given compound pathway problem. [CN, PS, V]	14.1 Student *A* wants to visit Student *B*. Roads are shown as lines on the grid. Only south and east travel directions can be used.  a) How many different paths can *A* take to get to *B*, if *A* has to travel along the lines that represent the roads? b) Change the middle square to a 2×2 grid, and repeat the question.

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General Outcomes	Specific Outcomes	Illustrative Examples
(continued)	P7–15. (SP19) Solve problems, using the binomial theorem where *N* belongs to the set of natural numbers. [CN, E, V]	15.1 Expand $(x + y)^7$, using the binomial theorem. 15.2 Find the 11th term of the expansion of $(x - 2)^{13}$. 15.3 Investigate the sample space for flipping 1 coin, 2 coins, 3 coins, 4 coins . . . , and make an organized list. Relate this organized list to Pascal's triangle and the binomial theorem. 15.4 Given a set of four elements, list the different proper and improper subsets, and organize them. How is this related to Pascal's triangle? How many subsets are there in total?

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General Outcomes	Specific Outcomes	Illustrative Examples
Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.	C6–6. (SP20) Construct a sample space for two or three events. [PS, R, V]	6.1 List the sample space for rolling a 6-sided die and flipping a coin. 6.2 Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
	C6–7. (SP21) Classify events as independent or dependent. [C]	7.1 Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement. 7.2 Sixty per cent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events?
	C6–8. (SP22) Solve problems, using the probabilities of mutually exclusive and complementary events. [CN, PS, R]	8.1 If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game? 8.2 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?

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General Outcomes	Specific Outcomes	Illustrative Examples
Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.	P7–16. (SP23) Determine the conditional probability of two events (Bayes' law). [E, PS, R]	16.1 In a particular country, the probability of a child being a girl is 0.510. A family of five children is known to have at least two girls. What is the probability of this family having exactly four girls? 16.2 It is known that 10% of a population has a certain disease. For a patient without the disease, a blood test for the disease gives a correct diagnosis 95% of the time. For a patient with the disease, the test gives a correct diagnosis 99% of the time. What is the probability that a person whose blood test shows the disease actually has the disease?
	P7–17. (SP24) Solve probability problems involving permutations, combinations and conditional probability. [E, PS, R]	17.1 Five books, each of a different colour, and including one red and one green book, are placed on a shelf. What is the probability of the red book being at one end and the green book at the other? 17.2 What is the probability of holding all four aces in a five-card hand dealt from a standard 52-card deck? 17.3 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot? c) What is the probability of Team A eventually winning? d) If Team B shot first, what is the probability of Team B eventually winning?
	P7–18. (SP25) Solve probability problems, using the binomial distribution as applied to small samples. [PS, R, T]	18.1 A written test for a driver's licence consists of 10 multiple-choice questions. To pass the test, a driver must answer 9 or 10 questions correctly. What is the probability of passing by guessing, if there are four possible answers to each question? 18.2 A family has nine children. Assuming that there is an equal likelihood for male and female births, what is the probability that there are seven boys and two girls? 18.3 An 8 km/h crash test was given to a sample of 20 cars. Four cars failed the test because of damaged bumpers. Find a 95% confidence interval for the proportion of cars that would fail this crash test.