

COURSES

derived from

The Common Curriculum Framework

for

K–12 MATHEMATICS

Grade 10 to Grade 12

Western Canadian Protocol for Collaboration in Basic Education

Call for Resources

JUNE 1996

MATHEMATICS 11

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The Common Curriculum Framework

for

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MATHEMATICS 11: GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY STRAND AND SUBSTRAND

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by strand and substrand, for the Mathematics 11 course.

The coding for mathematical processes follows the same scheme as in the *Common Curriculum Framework*.

CLUSTERS IN THE MATHEMATICS 11 COURSE

There are 5 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

Common clusters, numbered C4 to C5, are part of the mathematics expected of all students completing a K to 12 mathematics program.

Pure clusters, numbered P3 to P5, place more emphasis on precise mathematical theory. The approaches used are primarily algebraic and graphical.

CODING FOR ILLUSTRATIVE EXAMPLES (IEs)

The illustrative examples (IEs) listed on the following pages are organized by strand and substrand and have been correlated to specific outcomes (SOs). The numbers are taken directly from the *Common Curriculum Framework*.

NUMBERING SYSTEM

The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59 of the *Common Curriculum Framework*). For example, C2 – 6._(PR53) is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.

Mathematics 11

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

- | | |
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| [C] Communication | [PS] Problem Solving |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve consumer problems, using arithmetic operations.</p> <p><i>(continued)</i></p>	<p>C4–1. (N12) Solve consumer problems, including:</p> <ul style="list-style-type: none"> • wages earned in various situations • property taxation • exchange rates • unit prices. <p>[CN, E, PS, R, T]</p>	<p>1.1 Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota and graduated commission.</p> <p>1.2 Jane has a choice of two restaurants at which to work. Mario’s pays \$8/h, and tips average \$24 daily. Teppan’s pays \$5.50/h, and tips average \$35 daily. If Jane works 30 hours weekly, spread over four days, how much would she earn at each restaurant?</p> <p>1.3 Identify and calculate various payroll deductions, including income tax, CPP, UI, medical benefits, union and professional dues and life insurance premiums.</p> <p>1.4 Estimate, calculate and compare gross and net pay for various wage or salary earners in your community.</p> <p>1.5 The Ningart property has a market value of \$105 000. The assessed values in the area are 60% of market values. The tax rate is 32.3 mills of assessed value. What is the Ningarts’ monthly tax payment?</p> <p>1.6 The exchange rate on a given day in the United States is 28% and in Canada 38.8%. Explain why this is possible.</p> <p>1.7 A Canadian traveller goes from Switzerland to Germany. She knows that one Swiss franc is equivalent to \$1.26 Canadian (including exchange cost) and that one German mark is \$0.97 Canadian (including exchange cost). How many German marks does she get for 100 Swiss francs?</p> <p>1.8 Which provides better value for tomato soup, \$0.69 for 284 mL or \$1.79 for 907 mL?</p>

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<p><i>(continued)</i></p>	<p>C4–2. Reconcile financial statements including: (N13)</p> <ul style="list-style-type: none"> • cheque books with bank statements • cash register tallies with daily receipts. <p>[CN, PS, T]</p>	<p>2.1 The following petty cash transactions occurred during the first week of March.</p> <p>March 4 \$100 cheque was received to establish the fund. March 5 Bought \$12.50 worth of postage stamps. March 5 Spent \$10 to have something delivered by taxi. March 6 Spent \$6.50 for lunch. March 7 Paid a courier service \$25 for deliveries. March 7 Bought flowers for opening day, \$28. March 8 Replenished the fund by \$25. March 9 Postage stamps purchased for \$21.50.</p> <p>Determine if a final balance of \$20 is correct. If not, provide an explanation for the difference, and indicate possible ways to correct the problem.</p> <p>2.2 Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.</p> <table border="1" data-bbox="1225 1003 2424 1377"> <thead> <tr> <th>Month</th> <th>Previous Balance</th> <th>– Payment Made</th> <th>+ Purchases Charged</th> <th>⇒ Balance Due</th> <th>+ Credit Charges</th> <th>⇒ New Balance</th> </tr> </thead> <tbody> <tr> <td>February</td> <td>\$314.65</td> <td>\$100.00</td> <td>\$193.75</td> <td></td> <td>\$5.72</td> <td>\$414.12</td> </tr> <tr> <td>March</td> <td></td> <td>\$150.00</td> <td>\$ 59.60</td> <td></td> <td></td> <td></td> </tr> <tr> <td>April</td> <td></td> <td>\$140.00</td> <td>\$421.83</td> <td></td> <td></td> <td>\$618.62</td> </tr> <tr> <td>May</td> <td>\$618.62</td> <td>\$200.00</td> <td>\$ 39.65</td> <td></td> <td></td> <td></td> </tr> <tr> <td>June</td> <td></td> <td>\$250.00</td> <td>\$ 58.11</td> <td></td> <td></td> <td></td> </tr> <tr> <td>July</td> <td></td> <td>\$150.00</td> <td>\$ 77.21</td> <td></td> <td></td> <td></td> </tr> <tr> <td>August</td> <td>\$206.68</td> <td>\$120.00</td> <td>\$163.09</td> <td></td> <td>\$3.50</td> <td>\$253.27</td> </tr> </tbody> </table>	Month	Previous Balance	– Payment Made	+ Purchases Charged	⇒ Balance Due	+ Credit Charges	⇒ New Balance	February	\$314.65	\$100.00	\$193.75		\$5.72	\$414.12	March		\$150.00	\$ 59.60				April		\$140.00	\$421.83			\$618.62	May	\$618.62	\$200.00	\$ 39.65				June		\$250.00	\$ 58.11				July		\$150.00	\$ 77.21				August	\$206.68	\$120.00	\$163.09		\$3.50	\$253.27
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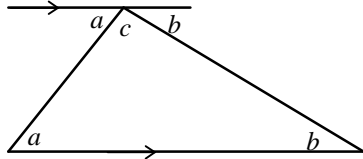
General Outcomes	Specific Outcomes	Illustrative Examples														
<i>(continued)</i>	<i>(continued)</i>	<p>4.2 Plot the world population on the vertical axis and the date on the horizontal axis. Use the graph to predict the date when the population reached 4 billion and to predict the present population of the world.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Date</th> <th>Population</th> </tr> </thead> <tbody> <tr><td>1650</td><td>500 000 000</td></tr> <tr><td>1850</td><td>1 100 000 000</td></tr> <tr><td>1930</td><td>2 000 000 000</td></tr> <tr><td>1950</td><td>2 500 000 000</td></tr> <tr><td>1970</td><td>3 600 000 000</td></tr> <tr><td>1988</td><td>5 100 000 000</td></tr> </tbody> </table> <p>C4–5. Solve investment and credit problems involving simple and compound interest. (N16) [CN, PS, T]</p> <p>5.1 Determine the effective annual interest rate on a loan of \$1000 at 10% per year, compounded quarterly.</p> <p>5.2 Calculate the compound amount, after one year, of a deposit of \$1000. Assume the current nominal annual interest when the interest is compounded:</p> <ol style="list-style-type: none"> annually monthly daily. <p>5.3 A bank offers an interest rate of 8% per year, compounded annually. A second bank offers an interest rate of 8% per year, compounded quarterly. If \$2000 were deposited, for ten years, in each bank, how much more income would be gained in the second bank than in the first?</p> <p>5.4 Calculate the interest paid on various forms of credit, including:</p> <ol style="list-style-type: none"> credit cards loans mortgages. <p>5.5 A loan of \$5000 carries an interest rate of 9% per year, compounded monthly. Adele makes a payment of \$350 every month. Use a spreadsheet to determine how much she still owes after making 12 payments.</p> <p>5.6 Compare two investments in an RRSP for one year with contributions starting January 1.</p> <ol style="list-style-type: none"> \$100 is invested monthly at 10% per annum, compounded monthly. \$600 is invested semi-annually at 10% per annum, compounded semi-annually. 	Date	Population	1650	500 000 000	1850	1 100 000 000	1930	2 000 000 000	1950	2 500 000 000	1970	3 600 000 000	1988	5 100 000 000
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Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Apply the principles of mathematical reasoning to solve problems and to justify solutions.</p> <p><i>(continued)</i></p>	<p>P5–1. Differentiate between inductive and deductive reasoning. [CN, R]</p> <p>P5–2. Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems. [C, PS, R, V]</p>	<p>1.1 Find, inductively, the sum of the angles of a triangle, by:</p> <ol style="list-style-type: none"> constructing triangles and tearing the corners off putting the torn corners together to form a straight line. <p>1.2 Show, deductively, that the sum of the measures a, b and c is 180°, by:</p> <ol style="list-style-type: none"> drawing a triangle using one side as a base and drawing a parallel line segment on the opposite vertex knowing that $a = a$, $b = b$, and c is included in both; $\therefore a + c + b = 180^\circ$.  <p>2.1 Each member of a sports club plays at least one of the following sports: soccer, rugby or tennis. The following information is given:</p> <ol style="list-style-type: none"> 163 play tennis; 36 play tennis and rugby; 13 play tennis and soccer 6 play all three sports; 11 play soccer and rugby; 208 play rugby or tennis 98 play soccer or rugby. <p>Use this information to determine the number of members in the club.</p> <p>2.2 On a number line, indicate the location of the sets corresponding to the following:</p> <ol style="list-style-type: none"> $x < 2$ or $x > 5$ $x < 2$ and $x > 5$ $x < 5$ or $x > 2$ $x < 5$ and not $x > 2$. <p>2.3 The phrase “A or B” can be used in ordinary speech in inclusive and exclusive senses, depending on whether “A and B” is included or excluded.</p> <ol style="list-style-type: none"> Give a practical example of each sense of “A or B”. Show the relationship between the inclusive and the exclusive sense of “A or B” on appropriate Venn diagrams. Mathematicians and logicians use the inclusive sense of “A or B”. Justify this choice.

Mathematics 11

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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P5–3. (PR7) Use examples and counterexamples to analyze conjectures. [CN, R]</p>	<p>3.1 Rajiv concluded that whenever he added two prime numbers the sum was always even. Find a counterexample to prove that Rajiv’s conjecture is false.</p> <p>3.2 A science text states that water boils at 100°C. Find a counterexample.</p> <p>3.3 Mary used her graphing calculator to graph $y = x^x$. She found the screen to be blank for $x < 0$ and made a conjecture that x^x is undefined when $x < 0$. Find an example to show that Mary’s conjecture is reasonable. Find a counterexample to show that Mary’s conjecture is false.</p> <p>3.4 The functions $f(x) = \frac{x^2 - 49}{x - 7}$ and $g(x) = x + 7$ are closely related.</p> <p>a) Explain the similarities and the differences between $f(x)$ and $g(x)$.</p> <p>b) How do the graphs of $f(x)$ and $g(x)$ differ from one another?</p>

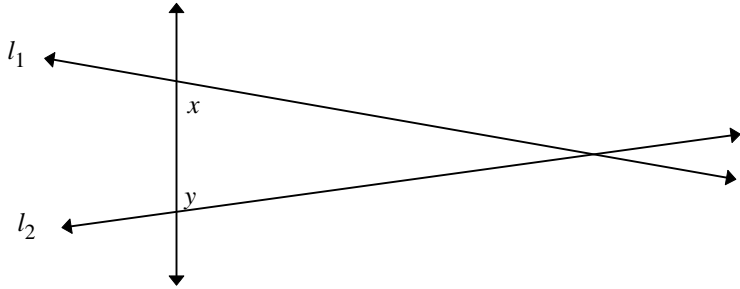
Mathematics 11

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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P5-4. (PR8) Distinguish between an “if-then” proposition, its converse and its contrapositive. [CN, R]</p> <p>P5-5. (PR9) Prove assertions in a variety of settings, using direct and indirect reasoning. [R]</p>	<p>4.1 Change the statement “Multiples of 3 are always multiples of 6” into “if-then” form, and write the converse and contrapositive of the “if-then” statement. Decide on the truth of all three propositions.</p> <p>4.2 Create a true proposition whose converse and contrapositive are both true.</p> <p>5.1 Angle ABC is obtuse, and AD is the median of BC. If AD is not an altitude, prove that ABC is a scalene triangle.</p> <p>5.2 Prove that the medians of a triangle cannot bisect each other.</p> <p>5.3 In the diagram below, show: a) $x + y < 180^\circ$ b) if $x + y = 180^\circ$, lines l_1 and l_2 are parallel.</p> <div style="text-align: center;">  </div> <p>5.4 Prove that the difference of squares of two odd numbers is always divisible by 4.</p>

Mathematics 11

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication

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Mental Mathematics

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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Represent and analyze situations that involve expressions, equations and inequalities.</p>	<p>C5-1. Graph linear inequalities, in two variables. [PS, V] (PR29)</p> <p>C5-2. Solve systems of linear equations, in two variables: • algebraically (elimination and substitution) • graphically. [CN, PS, T, V] (PR30)</p> <p>C5-3. Solve nonlinear equations, using a graphing tool. [CN, T, V] (PR31)</p>	<p>1.1 Solve, algebraically and graphically, for x: $2x + 5 > 3x - 1$.</p> <p>1.2 A target is described in terms of coordinates (x, y), where x and y are measured in metres. All of the following are true: • $x \leq 6$ • $y \geq 7$ • (x, y) is in the first quadrant • $x + y \leq 10$. What is the shape and the area of the target?</p> <p>2.1 Solve this system of equations, using the elimination method: $x + 2y = 10$ $2x + 3y = 14$.</p> <p>2.2 Solve this system of equations, using the substitution method: $3x + 4y = 15$ $x - y = 5$.</p> <p>2.3 A principal of \$42 000 is invested partly at 7% and partly at 9.5%. If the interest is \$3700, how much is invested at each interest rate?</p> <p>2.4 Plot the graphs of $2x + 3y = 11$ and $2x - 3y = 17$. What is their point of intersection?</p> <p>3.1 Using a graphing tool, solve $x^2 + 6x - 11 = 0$.</p> <p>3.2 Solve $x^3 + x = 30$ graphically, using two different methods. Which method gives solutions that are freer from rounding errors and other inaccuracies?</p> <p>3.3 Where does the line $y = 4x + 5$ cut the curve $y = 2^x$? Use a graphing tool to find the points of intersection.</p>

Mathematics 11

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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Represent and analyze situations that involve expressions, equations and inequalities.</p> <p><i>(continued)</i></p>	<p>P3-1. Solve nonlinear equations: (PR32)</p> <ul style="list-style-type: none"> by factoring graphically. <p>[CN, T, V]</p> <p>P3-2. Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. (PR33)</p> <p>[E, PS, T]</p>	<p>1.1 Solve by factoring:</p> <ol style="list-style-type: none"> $x^2 - 2x = 24$ $x^3 = 1$ $2x^2 + 9x - 5 = 0$ $7x^2 + 4x - 11 = 0$. <p>1.2 Solve each of the above graphically. For example, $x^2 - 2x = 24$ can be solved by graphing $y = x^2 - 2x$ and $y = 24$ and using the points of intersection to determine the solution.</p> <p>1.3 Solve $3x^2 + 1 = 10x - 2$ graphically in two different ways. Is there one way that gives more reliable results? Explain your procedures and the results obtained.</p> <p>2.1 The polynomial $P(x) = 4x^3 + bx^2 + cx + 11$ has a remainder of -7 when divided by $(x + 2)$ and a remainder of 14 when divided by $(x - 1)$. Find the values of b and c.</p> <p>2.2 Factor $x^3 - 2x^2 - 5x + 6$.</p>

Mathematics 11

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<p><i>(continued)</i></p>	<p>P3–3. (PR34) Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]</p>	<p>3.1 Find the solutions to the following system: $y = x^2$ $y = 8 - x^2$.</p> <p>3.2 Graphically, find the solution set to the following system: $y = 3x + 2$ and $y = 2^x$.</p> <p>How do you know that the solution set is complete?</p> <p>3.3 The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. The population growth can be modelled by the equation $P_1 = 5(1.02)^n$, with the food production being modelled by $P_2 = 0.2n + 6$. The variable n is the number of years after 1987.</p> <p>a) When does $P_1 = P_2$?</p> <p>b) If $P_1 > P_2$ is true, when does this happen, and how is this inequality interpreted?</p>

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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P3–4. Solve systems of linear equations, in three variables: (PR35)</p> <ul style="list-style-type: none"> algebraically with technology. <p>[CN, PS, T, V]</p>	<p>4.1 Determine the solution to the following system: $2x + y - z = 3$ $x + 2y + z = 0$ $3x - y - 2z = 11.$</p> <p>4.2 The total revenue R is a quadratic function of the price p of books sold. So $R = ap^2 + bp + c$. Find the values of a, b and c, if the revenue is \$6000 at a price of \$30, \$6000 at a price of \$40 and \$5000 at a price of \$50.</p>

Mathematics 11

Strand: Patterns and Relations (Relations and Functions)

Students will:

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<p>Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.</p>	<p>C5-4. (PR57) Determine the following characteristics of the graph of a quadratic function:</p> <ul style="list-style-type: none"> • vertex • domain and range • axis of symmetry • intercepts. <p>[C, PS, T, V]</p>	<p>4.1 Given the graph of any quadratic function, determine the following:</p> <ol style="list-style-type: none"> vertex domain range axis of symmetry intercepts. <p>4.2 Use technology to graph $f(x) = x^2 - 6x + 4$ and to determine the vertex, domain, range, axis of symmetry and intercepts.</p> <p>4.3 One model concerning the rate of population growth of Earth has the annual rate of increase varying jointly as the population and the unused carrying capacity of Earth. The equation of the model is: $y = 0.001x(21 - x)$, where y = the rate of increase in population (in billions per year), and x = the present population (in billions).</p> <ol style="list-style-type: none"> Plot this model of growth. The present population of Earth is 5.8 billion. What is the annual increase in population at present? What is the population when the rate of increase in population is at its greatest? What is the population when the rate of increase is zero? What is the projected maximum population that Earth can accommodate, according to this model?

Mathematics 11

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<p>Examine the nature of relations with an emphasis on functions.</p> <p><i>(continued)</i></p>	<p>P4–1. Perform operations on functions and compositions of functions. [CN, E, PS]</p>	<p>1.1 If $f(x) = 3x + 2$ and $g(x) = x^2$, find:</p> <ol style="list-style-type: none"> $3f(x)$ $f(x) \cdot g(x)$ $f(x) + g(x)$ $f(g(x))$ $f(f(x))$. <p>1.2 A ball thrown in the air has a velocity given by $v(t) = 49 - 9.8t$. The kinetic energy function $K(v)$ is given by $K(v) = 0.4v^2$. Express the ball's kinetic energy as a function $K(t)$ of time.</p>

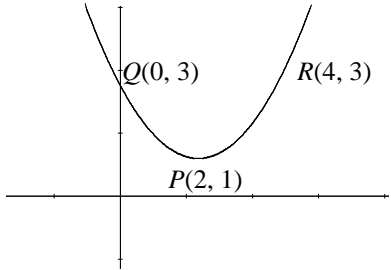
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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P4–2. Determine the inverse of a function. (PR55) [CN, R, V]</p>	<p>2.1 Graph the inverse of $y = \frac{x}{(x-1)}$, and determine the equation, domain and range of the inverse.</p> <p>2.2 Sketch the inverse of the following.</p> <div style="text-align: center;">  <p>The diagram shows a Cartesian coordinate system with a parabola opening upwards. The vertex is labeled P(2, 1). Two other points on the parabola are labeled Q(0, 3) and R(4, 3). The x-axis and y-axis are shown with tick marks.</p> </div> <p>2.3 Sketch the inverse of $f(x) = x^2$.</p> <p>2.4 If $f(x) = 2x - 1$ and $g(x) = \frac{x+1}{2}$, find $f(g(x))$ and $g(f(x))$, and show that the functions $f(x)$ and $g(x)$ are inverses of each other.</p> <p>2.5 Determine the domain and range for each of the functions in illustrative examples 9.2 and 9.3.</p> <p>2.6 Is the inverse of $f(x) = 2x - 5$ a function?</p>

Mathematics 11

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication

[CN] Connections

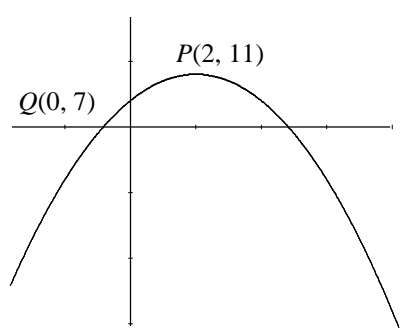
[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.</p> <p><i>(continued)</i></p>	<p>P4–3. (PR58) Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]</p>	<p>3.1 Graph $f(x) = 2x^2 + 5x - 7$.</p> <p>3.2 Give a list of events or situations that might be described by a quadratic, parabolic, shape.</p> <p>3.3 Given the graph of $y = x^2$, sketch $y = -2(x - 3)^2 - 4$.</p> <p>3.4 Given the graph of $y = x^2$, what is the equation for the transformed graph shown here?</p>  <p>3.5 Rewrite the equation of $f(x) = 2x^2 - 12x + 13$ in the form $f(x) = a(x - p)^2 + q$, and graph the function.</p>
	<p>P4–4. (PR59) Model real-world situations, using quadratic functions. [CN, PS]</p>	<p>4.1 Computer software programs are sold to students for \$20 each, and 300 students are willing to buy them at that price. For every \$5 increase in price, there are 30 fewer students willing to buy the software. What is the maximum revenue?</p> <p>4.2 What is the maximum rectangular area that can be enclosed by 120 m of fencing, if one of the sides of the rectangle is an existing wall?</p>

Mathematics 11

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

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| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
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Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<i>(continued)</i>	<p>P4–5. (PR60) Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:</p> <ul style="list-style-type: none"> • factoring • the quadratic formula • graphing. <p>[CN, E, T, V]</p>	<p>5.1 Solve $3x^2 - 5x + 2 = 0$ algebraically and by graphing the corresponding function $f(x) = 3x^2 - 5x + 2$.</p> <p>5.2 When bicycles are sold for \$280 each, a cycle store can sell 80 in a season. For every \$10 increase in the price, the number sold drops by 3.</p> <ol style="list-style-type: none"> Represent the sales revenue as a quadratic function of either the number sold or the price. What is the number sold, and the price, if the total sales revenue is exactly \$20 000? What is the range of prices that will give a sales revenue that exceeds \$15 000? <p>5.3 Write a quadratic equation whose roots are $\frac{3}{2}$ and $-\frac{1}{4}$. Is this equation unique?</p>

Mathematics 11

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

- | | |
|--|----------------------|
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Mental Mathematics | [T] Technology |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P4–6. (PR61) Determine the character of the real and non-real roots of a quadratic equation, using:</p> <ul style="list-style-type: none"> • the discriminant in the quadratic formula • graphing. <p>[C, R, T, V]</p>	<p>6.1 If $3x^2 - mx + 2 = 0$ can be factored, what values of m are possible?</p> <p>6.2 Discuss the implications of a negative discriminant when describing the zeros of a quadratic function.</p> <p>6.3 Given $3x^2 - mx + 3 = 0$:</p> <ol style="list-style-type: none"> For what value(s) of m would one root be double the other? For what values of m would the roots not be real? <p>6.4 The profit y for publishing a book is given by the equation $y = -5x^2 + 400x - 3000$, where x is the selling price per book.</p> <ol style="list-style-type: none"> Is it possible to set a selling price that will earn a total profit of \$6000? Explain your solution with reference to appropriate equations and graphs. What range of selling prices allow the publisher to make a profit on this book?

Mathematics 11

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication

[CN] Connections

[E] Estimation and

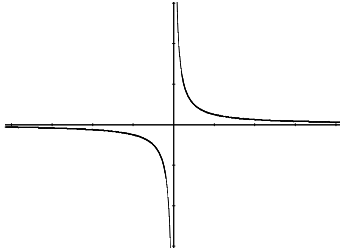
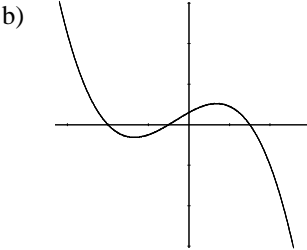
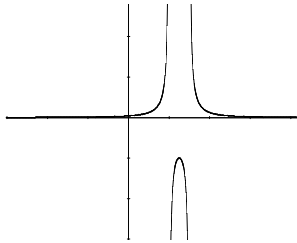
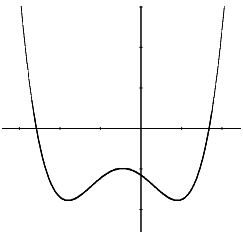
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P4–7. Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]</p> <p style="text-align: right;"><i>(continued)</i></p>	<p>7.1 Determine if each of the following examples is a rational function, a polynomial function or some other type of function, and justify your conclusion.</p> <p>a) $y = x^2 - 3x + \sqrt{7}$</p> <p>b) $y = (x - 5)^{-1}$</p> <p>c) $y = \frac{1}{5}x^4 + 3x^3 - 12x - 0.75$</p> <p>d) $y = \sqrt{7x^5} + x^2$</p> <p>e) $y = 2^x - 9$</p> <p>f) $y = \frac{3x - 7}{x^2 - 5x + 6}$</p> <p>7.2 Examine the following graphs. Which could be graphs of rational functions, and which could be graphs of polynomial functions?</p> <p>a) </p> <p>b) </p> <p>c) </p> <p>d) </p>

Mathematics 11

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

[C] Communication

[CN] Connections

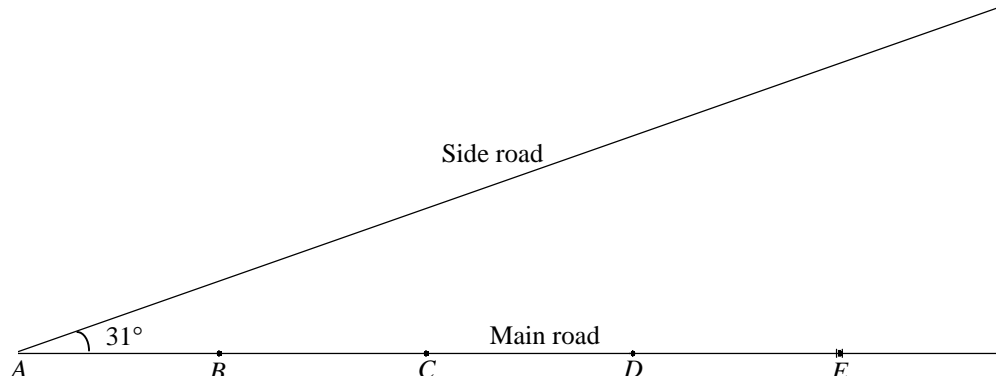
[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve problems involving triangles, including those found in 3-D and 2-D applications.</p>	<p>P3–5. (SS7) Solve problems involving ambiguous case triangles in 3-D and 2-D. [CN, PS, R, T]</p>	<p>5.1 An 11 cm long line AB is drawn at an angle of 44° to a horizontal line AE. A circle with centre B and a radius of 9 cm is drawn, cutting the horizontal line at points C and D. Calculate the length of the chord CD.</p> <p>5.2 The line segment of equation $y = 2.4x$, passes through $A(0, 0)$ and $C(5, 12)$, has a length of 13 and makes an angle of 67.3° with the horizontal x-axis.</p> <p>a) What points are located with $CB = 10$ and AB horizontal?</p> <p>b) Check your answer by determining the intersection points of the circle $(x - 5)^2 + (y - 12)^2 = 100$ and the line $y = 0$.</p> <p>c) Use a suitable diagram to explain why the answers to a) and b) are the same.</p> <p>5.3</p>  <p>Streetlights A, B, C, D and E are placed 50 m apart on the main road, as indicated on the diagram. The light from a streetlight can travel 24 m. Determine the furthest point on the side road that is lighted and the length of side road that is illuminated by both streetlight C and streetlight D.</p>

Mathematics 11

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve coordinate geometry problems involving lines and line segments, and justify the solutions.</p>	<p>P3-6. Solve problems involving distances between points and lines. [CN, PS, R]</p> <p>P3-7. Verify and prove assertions in plane geometry, using coordinate geometry. [C, R, V]</p>	<p>6.1 Determine the shortest distance from (3, 4) to the line $2x - 5y = 7$.</p> <p>6.2 The lines $y = 3x + 1$ and $y = 3x - 9$ are parallel. Determine the vertical distance between the two lines, the horizontal distance between the two lines and the shortest distance between the two lines.</p> <p>7.1 Given $A = (-1, 3)$, $B = (0, 5)$ and $C = (-2, 6)$:</p> <ol style="list-style-type: none"> Verify that ABC is a right-angled triangle. Is ABC isosceles? Justify your assertion. If M is the midpoint of AB and N is the midpoint of AC, prove that MN is parallel to BC. Find a point D so that $ABCD$ is a parallelogram. Prove that $ABCD$ is not a rectangle. <p>7.2 Use coordinate geometry to prove that:</p> <ol style="list-style-type: none"> the diagonals of any parallelogram bisect one another if ABC is any triangle, with M as the midpoint of AB and N as the midpoint of AC, then MN is parallel to BC and is half its length. <p>7.3 Use coordinate geometry to divide the line segment with end points $A(4, 7)$ and $B(-3, 8)$ into five congruent parts.</p>

Mathematics 11

Strand: Shape and Space (3-D Objects and 2-D Shapes)

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[C] Communication

[CN] Connections

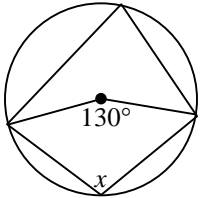
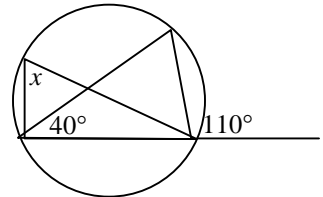
[E] Estimation and
Mental Mathematics

[PS] Problem Solving

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[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Develop and apply the geometric properties of circles and polygons to solve problems.</p> <p style="text-align: right;"><i>(continued)</i></p>	<p>C5-5. (SS26) Use technology and measurement to confirm and apply the following properties to particular cases:</p> <ul style="list-style-type: none"> the perpendicular from the centre of a circle to a chord bisects the chord the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc the inscribed angles subtended by the same arc are congruent the angle inscribed in a semicircle is a right angle the opposite angles of a cyclic quadrilateral are supplementary a tangent to a circle is perpendicular to the radius at the point of tangency the tangent segments to a circle, from any external point, are congruent the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles. <p>[PS, R, T, V]</p> <p style="text-align: right;"><i>(continued)</i></p>	<p>5.1 A plate, with a diameter of 20 cm, is placed on a square place mat, with no overhang. Calculate the length of the diagonal of the square.</p> <p>5.2 Determine the measure of angle x.</p>  <p>5.3 Determine the measure of angle x.</p>  <p>5.4 Draw a semicircle with diameter AB. Draw an angle, ACB, with C being any point on the semicircle. What is the measure of angle ACB? Repeat for two other points, C' and C'', on the semicircle. What pattern emerges?</p>

Mathematics 11

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication

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[E] Estimation and

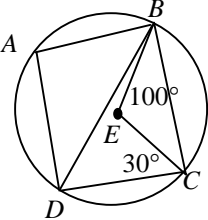
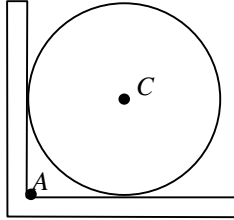
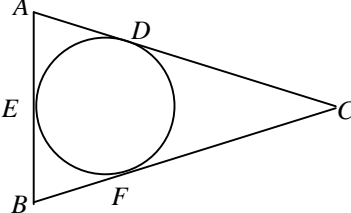
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p><i>(continued)</i></p>	<p>5.5 Determine the measure of $\angle ECB$, $\angle BDC$, $\angle BAD$ and $\angle DBE$, where E is the centre of the circle.</p>  <p>5.6 How far from the inside corner of the shelf, A, is the centre C of the plate, if the plate has a diameter of 20 cm?</p>  <p>5.7 The perimeter of the isosceles triangle ABC, with $AC = BC$, is 54 cm. If $AD = 5$ cm, and D, E and F are points of tangency, find the length of BC.</p> 

Mathematics 11

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

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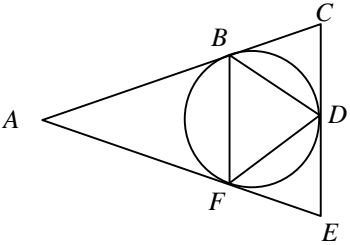
[E] Estimation and
Mental Mathematics

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[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<i>(continued)</i>	<i>(continued)</i>	<p>5.8 Determine the measure of $\angle CAE$, if $\angle BDF = 60^\circ$ and $\angle FDE = 70^\circ$.</p> 

Mathematics 11

Strand: Shape and Space (3-D Objects and 2-D Shapes)
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| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Develop and apply the geometric properties of circles and polygons to solve problems.</p> <p><i>(continued)</i></p>	<p>P5–6. (SS28) Prove the following general properties, using established concepts and theorems:</p> <ul style="list-style-type: none"> the perpendicular bisector of a chord contains the centre of the circle the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle) the inscribed angles subtended by the same arc are congruent the angle inscribed in a semicircle is a right angle the opposite angles of a cyclic quadrilateral are supplementary a tangent to a circle is perpendicular to the radius at the point of tangency the tangent segments to a circle from any external point are congruent the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles. <p>[C, R, V]</p>	<p>6.1 a) For what values of c does the line $y = c$ touch the circle $x^2 + y^2 = r^2$? b) Use the result from part a) to show that the tangent to a circle is perpendicular to the radius at the point of tangency.</p> <p>6.2 Show that the angle inscribed in a semicircle is a right angle.</p> <p>6.3 The chord AB is one side of a regular polygon of n sides. The polygon is inscribed in a circle. If D is any other vertex of the polygon, prove that the magnitude of angle ADB is $\frac{180^\circ}{n}$.</p>

Mathematics 11

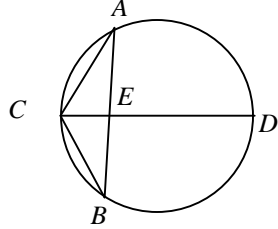
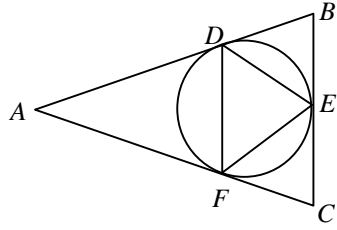
Strand: Shape and Space (3-D Objects and 2-D Shapes)

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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P5–7. (SS29) Solve problems, using a variety of circle properties, and justify the solution strategy used. [PS, R, V]</p>	<p>7.1 If diameter CD is perpendicular to chord AB at E, prove that triangle ABC is isosceles.</p> <div style="text-align: center;">  </div> <p>7.2 Determine the measure of $\angle BAC$, if $\angle DEF = 60^\circ$ and $\angle EFC = 70^\circ$. Provide a reason for each step in the solution strategy.</p> <div style="text-align: center;">  </div> <p>7.3 A chain on a bicycle connects two gear wheels of diameters 9 cm and 19 cm respectively. The centres of the gear wheels are 87 cm apart. Find the minimum length of the chain.</p>