The Common Curriculum Framework

for

GRADES 10–12 MATHEMATICS

Western and Northern Canadian Protocol

January 2008

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BACKGROUND

The Western Canadian Protocol (WCP) for Collaboration in Basic Education Kindergarten to Grade 12 was signed in December 1993 by the Ministers of Education from Alberta, British Columbia, Manitoba, Northwest Territories, Saskatchewan and Yukon Territory. In February 2000, following the addition of Nunavut, the protocol was renamed the Western and Northern Canadian Protocol (WNCP) for Collaboration in Basic Education (Kindergarten to Grade 12).

WNCP jurisdictions:

Alberta
British Columbia
Manitoba
Northwest Territories
Nunavut
Saskatchewan
Yukon Territory

In 2005, the Ministers of Education from all the WNCP jurisdictions unanimously concurred with the rationale of the original partnership because of the importance placed on:

- common educational goals
- the ability to collaborate to achieve common goals
- high standards in education
- planning an array of educational opportunities
- removing obstacles to accessibility for individual learners
- the optimum use of limited educational resources.

The original WCP Common Curriculum Framework for Mathematics was published in two documents: Kindergarten to Grade 9 in 1995 and Grades 10–12 in 1996.

The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol was developed by the seven ministries of education in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others.

The framework identifies beliefs about mathematics learning and teaching, general and specific outcomes, and achievement indicators agreed upon by the seven jurisdictions. Each of the provinces and territories will determine when and how the framework will be implemented within its own jurisdiction.

INTRODUCTION

PURPOSE OF THE DOCUMENT

The framework communicates high expectations for students' mathematical learnings.

This document provides sets of outcomes to be used as a common base for defining mathematics curriculum expectations that will be mandated by each province and territory in grades 10, 11 and 12. This common base should result in consistent student outcomes in mathematics across the WNCP jurisdictions and enable easier transfer for students moving from one jurisdiction to another. This document is intended to clearly communicate high expectations for students' mathematical learnings in grades 10, 11 and 12 to all education partners across the jurisdictions, and to facilitate the development of common learning resources.

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

Mathematical understanding is fostered when students build on their own experiences and prior knowledge.

Students are curious, active learners with individual interests, abilities, needs and career goals. They come to school with varying knowledge, life experiences, expectations and backgrounds. A key component in developing mathematical literacy in students is making connections to these backgrounds, experiences, goals and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics.

The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

FIRST NATIONS, MÉTIS AND INUIT PERSPECTIVES

First Nations, Métis and Inuit students in northern and western Canada come from diverse geographic areas and have varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

Teachers need to understand the diversity of students' cultures and experiences.

First Nations, Métis and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning and learn mathematics best when it is contextualized and not taught as discrete content.

First Nations, Métis and Inuit students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to nonverbal cues so that student learning and mathematical understanding are optimized.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students.

The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

AFFECTIVE DOMAIN

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, support risk taking and provide opportunities for success help students to develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, to participate willingly in classroom activities, to persist in challenging situations and to engage in reflective practices.

To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains and to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Curiosity about mathematics is fostered when students are actively engaged in their environment.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

GOALS FOR STUDENTS

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

- solve problems
- communicate and reason mathematically
- make connections between mathematics and its applications
- become mathematically literate
- appreciate and value mathematics
- make informed decisions as contributors to society.

Students who have met these goals:

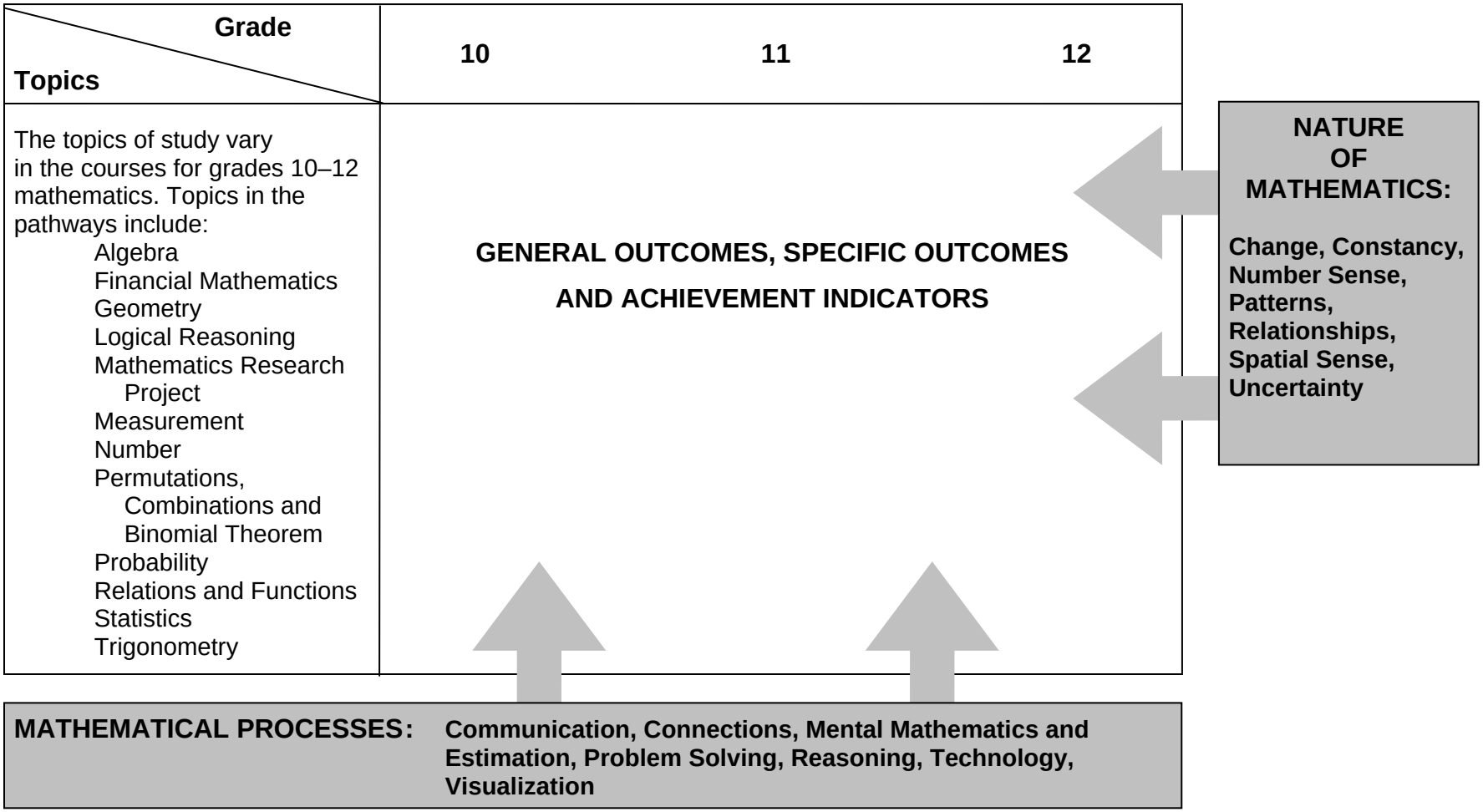
- gain an understanding and appreciation of the role of mathematics in society
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical problem solving
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics.

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

CONCEPTUAL FRAMEWORK FOR GRADES 10–12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



MATHEMATICAL PROCESSES

The seven mathematical processes are critical aspects of learning, doing and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education.

The common curriculum framework incorporates the following interrelated mathematical processes. It is intended that they permeate the teaching and learning of mathematics.

Students are expected to:

- *Communication [C]* — use *communication* in order to learn and express their understanding
- *Connections [CN]* — make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
- *Mental Mathematics and Estimation [ME]* — demonstrate fluency with *mental mathematics and estimation*
- *Problem Solving [PS]* — develop and apply new mathematical knowledge through *problem solving*
- *Reasoning [R]* — develop mathematical *reasoning*
- *Technology [T]* — select and use *technology* as a tool for learning and solving problems
- *Visualization [V]* — develop *visualization* skills to assist in processing information, making connections and solving problems.

All seven processes should be used in the teaching and learning of mathematics. Each specific outcome includes a list of relevant mathematical processes. The identified processes are to be used as a primary focus of instruction and assessment.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics.

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can play a significant role in helping students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Through connections, students begin to view mathematics as useful and relevant.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding.… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves using strategies to perform mental calculations.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility in reasoning and calculating.

Mental mathematics and estimation are fundamental components of number sense.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities, usually by referring to benchmarks or referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgements and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

Learning through problem solving should be the focus of mathematics at all grade levels.

When students encounter new situations and respond to questions of the type, *How would you ...?* or *How could you ...?,* the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks.

Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ….*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

The use of technology should not replace mathematical understanding.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

NATURE OF MATHEMATICS

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of understanding, interpreting and describing our world. There are a number of characteristics that define the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Change is an integral part of mathematics and the learning of mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

Some problems in mathematics require students to focus on properties that remain constant.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations.

Number Sense

Number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics. Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment.

Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems.

Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships.

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data inherently lack certainty.

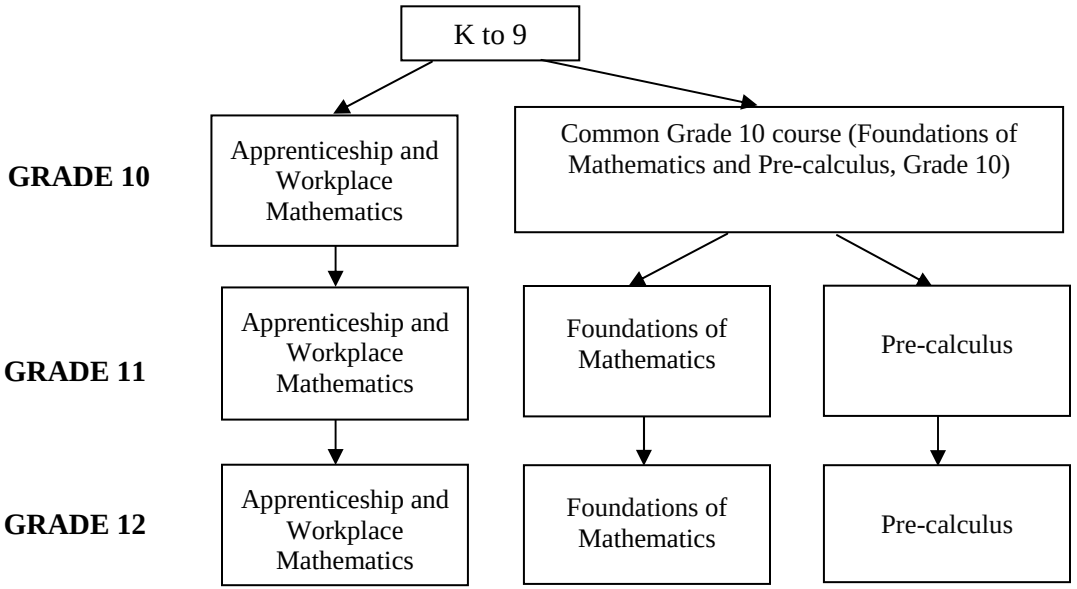
Events and experiments generate statistical data that can be used to make predictions. It is important that students recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of an interpretation or conclusion is directly related to the quality of the data it is based upon. An awareness of uncertainty provides students with an understanding of why and how to assess the reliability of data and data interpretation.

Uncertainty is an inherent part of making predictions.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

PATHWAYS AND TOPICS

The Common Curriculum Framework for Grades 10–12 Mathematics includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. Three pathways are available: Apprenticeship and Workplace Mathematics, Foundations of Mathematics, and Pre-calculus. A common Grade 10 course (Foundations of Mathematics and Pre-calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.



Goals of Pathways

The goals of all three pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Apprenticeship and Workplace Mathematics
This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics and probability.

Foundations of Mathematics
This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, statistics and probability.

Pre-calculus
This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, and permutations, combinations and binomial theorem.

OUTCOMES AND ACHIEVEMENT INDICATORS

The common curriculum framework is stated in terms of general outcomes, specific outcomes and achievement indicators.

General outcomes are overarching statements about what students are expected to learn in each course.

Specific outcomes are statements that identify the specific knowledge, skills and understandings that students are required to attain by the end of a given course.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.

The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question. The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.

SUMMARY

The Conceptual Framework for Grades 10–12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in grades 10–12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

INSTRUCTIONAL FOCUS

Each pathway in *The Common Curriculum Framework for Grades 10–12 Mathematics* is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful.

Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

APPRENTICESHIP AND WORKPLACE MATHEMATICS GRADE 10

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement	**General Outcome:** Develop spatial sense through direct and indirect measurement.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • applying strategies to convert SI units to imperial units. [C, CN, ME, V]	*(It is intended that this outcome be limited to the base units and the prefixes milli, centi, deci, deca, hecto and kilo.)* 1.1 Explain how the SI system was developed, and explain its relationship to base ten. 1.2 Identify the base units of measurement in the SI system, and determine the relationship among the related units of each type of measurement. 1.3 Identify contexts that involve the SI system. 1.4 Match the prefixes used for SI units of measurement with the powers of ten. 1.5 Explain, using examples, how and why decimals are used in the SI system. 1.6 Provide an approximate measurement in SI units for a measurement given in imperial units; e.g., 1 inch is approximately 2.5 cm. 1.7 Write a given linear measurement expressed in one SI unit in another SI unit. 1.8 Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas); e.g., Celsius to Fahrenheit, centimetres to inches.
2. Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • comparing the American and British imperial units for capacity • applying strategies to convert imperial units to SI units. [C, CN, ME, V]	2.1 Explain how the imperial system was developed. 2.2 Identify commonly used units in the imperial system, and determine the relationships among the related units. 2.3 Identify contexts that involve the imperial system. 2.4 Explain, using examples, how and why fractions are used in the imperial system. 2.5 Compare the American and British imperial measurement systems; e.g., gallons, bushels, tons. 2.6 Provide an approximate measure in imperial units for a measurement given in SI units; e.g., 1 litre is approximately 1/4 US gallon. 2.7 Write a given linear measurement expressed in one imperial unit in another imperial unit. 2.8 Convert a given measure from imperial to SI units by using proportional reasoning (including formulas); e.g., Fahrenheit to Celsius, inches to centimetres.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement (continued)	**General Outcome:** Develop spatial sense through direct and indirect measurement.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V]	*(It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.)* 3.1 Identify a referent for a given common SI or imperial unit of linear measurement. 3.2 Estimate a linear measurement, using a referent. 3.3 Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments. 3.4 Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent; e.g., the height of the desk is about three rulers long, so the desk is approximately three feet high. 3.5 Solve a linear measurement problem including perimeter, circumference, and length + width + height (used in shipping and air travel). 3.6 Determine the operation that should be used to solve a linear measurement problem. 3.7 Provide an example of a situation in which a fractional linear measurement would be divided by a fraction. 3.8 Determine, using a variety of strategies, the midpoint of a linear measurement such as length, width, height, depth, diagonal and diameter of a 3-D object, and explain the strategies. 3.9 Determine if a solution to a problem that involves linear measurement is reasonable.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement (continued)	**General Outcome:** Develop spatial sense through direct and indirect measurement.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]	*(It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.)* 4.1 Identify and compare referents for area measurements in SI and imperial units. 4.2 Estimate an area measurement, using a referent. 4.3 Identify a situation where a given SI or imperial area unit would be used. 4.4 Estimate the area of a given regular, composite or irregular 2-D shape, using an SI square grid and an imperial square grid. 4.5 Solve a contextual problem that involves the area of a regular, a composite or an irregular 2-D shape. 4.6 Write a given area measurement expressed in one SI unit squared in another SI unit squared. 4.7 Write a given area measurement expressed in one imperial unit squared in another imperial unit squared. 4.8 Solve a problem, using formulas for determining the areas of regular, composite and irregular 2-D shapes, including circles. 4.9 Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones. 4.10 Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles. 4.11 Determine if a solution to a problem that involves an area measurement is reasonable.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	*(It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.)* 1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. 1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. 1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.
2. Demonstrate an understanding of the Pythagorean theorem by: • identifying situations that involve right triangles • verifying the formula • applying the formula • solving problems. [C, CN, PS, V]	2.1 Explain, using illustrations, why the Pythagorean theorem only applies to right triangles. 2.2 Verify the Pythagorean theorem, using examples and counterexamples, including drawings, concrete materials and technology. 2.3 Describe historical and contemporary applications of the Pythagorean theorem. 2.4 Determine if a given triangle is a right triangle, using the Pythagorean theorem. 2.5 Explain why a triangle with the side length ratio of 3:4:5 is a right triangle. 2.6 Explain how the ratio of 3:4:5 can be used to determine if a corner of a given 3-D object is square (90º) or if a given parallelogram is a rectangle. 2.7 Solve a problem, using the Pythagorean theorem.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry (continued)	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V]	3.1 Determine, using angle measurements, if two or more regular or irregular polygons are similar. 3.2 Determine, using ratios of side lengths, if two or more regular or irregular polygons are similar. 3.3 Explain why two given polygons are not similar. 3.4 Explain the relationships between the corresponding sides of two polygons that have corresponding angles of equal measure. 3.5 Draw a polygon that is similar to a given polygon. 3.6 Explain why two or more right triangles with a shared acute angle are similar. 3.7 Solve a contextual problem that involves similarity of polygons.
4. Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles • generalizing patterns from similar right triangles • applying the primary trigonometric ratios • solving problems. [CN, PS, R, T, V]	4.1 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio. 4.2 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio. 4.3 Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio. 4.4 Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths. 4.5 Solve a contextual problem that involves right triangles, using the primary trigonometric ratios. 4.6 Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry (continued)	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
5. Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]	5.1 Sort a set of lines as perpendicular, parallel or neither, and justify this sorting. 5.2 Illustrate and describe complementary and supplementary angles. 5.3 Identify, in a set of angles, adjacent angles that are not complementary or supplementary. 5.4 Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on same side of transversal and exterior angles on same side of transversal. 5.5 Explain and illustrate the relationships of angles formed by parallel lines and a transversal. 5.6 Explain, using examples, why the angle relationships do not apply when the lines are not parallel. 5.7 Determine if lines or planes are perpendicular or parallel, e.g., wall perpendicular to floor, and describe the strategy used. 5.8 Determine the measures of angles involving parallel lines and a transversal, using angle relationships. 5.9 Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).
6. Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: • drawing • replicating and constructing • bisecting • solving problems. [C, ME, PS, T, V]	6.1 Draw and describe angles with various measures, including acute, right, straight, obtuse and reflex angles. 6.2 Identify referents for angles. 6.3 Sketch a given angle. 6.4 Estimate the measure of a given angle, using 22.5°, 30°, 45°, 60°, 90° and 180° as referent angles. 6.5 Measure, using a protractor, angles in various orientations. 6.6 Explain and illustrate how angles can be replicated in a variety of ways; e.g., Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software. 6.7 Replicate angles in a variety of ways, with and without technology. 6.8 Bisect an angle, using a variety of methods. 6.9 Solve a contextual problem that involves angles.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]	1.1 Compare the unit price of two or more given items. 1.2 Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity. 1.3 Compare, using examples, different sales promotion techniques; e.g., deli meat at $2 per 100 g seems less expensive than $20 per kilogram. 1.4 Determine the percent increase or decrease for a given original and new price. 1.5 Solve, using proportional reasoning, a contextual problem that involves currency exchange. 1.6 Explain the difference between the selling rate and purchasing rate for currency exchange. 1.7 Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important. 1.8 Convert between Canadian currency and foreign currencies, using formulas, charts or tables.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number (continued)	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Demonstrate an understanding of income, including: • wages • salary • contracts • commissions • piecework to calculate gross pay and net pay. [C, CN, R, T]	2.1 Describe, using examples, various methods of earning income. 2.2 Identify and list jobs that commonly use different methods of earning income; e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums. 2.3 Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time. 2.4 Determine gross pay from given or calculated hours worked when given: • the base hourly wage, with and without tips • the base hourly wage, plus overtime (time and a half, double time). 2.5 Determine gross pay for earnings acquired by: • base wage, plus commission • single commission rate. 2.6 Explain why gross pay and net pay are not the same. 2.7 Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay. 2.8 Determine net pay when given deductions; e.g., health plans, uniforms, union dues, charitable donations, payroll tax. 2.9 Investigate, with technology, "what if …" questions related to changes in income; e.g., "What if there is a change in the rate of pay?" 2.10 Identify and correct errors in a solution to a problem that involves gross or net pay. 2.11 Describe the advantages and disadvantages for a given method of earning income; e.g., hourly wage, tips, piecework, salary, commission, contract work.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra	**General Outcome:** Develop algebraic reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	*(It is intended that this outcome be integrated throughout the course.)* 1.1 Solve a contextual problem that involves the application of a formula that does not require manipulation. 1.2 Solve a contextual problem that involves the application of a formula that requires manipulation. 1.3 Explain and verify why different forms of the same formula are equivalent. 1.4 Describe, using examples, how a given formula is used in a trade or an occupation. 1.5 Create and solve a contextual problem that involves a formula. 1.6 Identify and correct errors in a solution to a problem that involves a formula.

APPRENTICESHIP AND WORKPLACE MATHEMATICS GRADE 11

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement	**General Outcome:** Develop spatial sense through direct and indirect measurement.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve SI and imperial units in surface area measurements and verify the solutions. [C, CN, ME, PS, V]	1.1 Explain, using examples, the difference between volume and surface area. 1.2 Explain, using examples, including nets, the relationship between area and surface area. 1.3 Explain how a referent can be used to estimate surface area. 1.4 Estimate the surface area of a 3-D object. 1.5 Illustrate, using examples, the effect of dimensional changes on surface area. 1.6 Solve a contextual problem that involves the surface area of 3-D objects, including spheres, and that requires the manipulation of formulas.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement (continued)	**General Outcome:** Develop spatial sense through direct and indirect measurement.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V]	2.1 Explain, using examples, the difference between volume and capacity. 2.2 Identify and compare referents for volume and capacity measurements in SI and imperial units. 2.3 Estimate the volume or capacity of a 3-D object or container, using a referent. 2.4 Identify a situation where a given SI or imperial volume unit would be used. 2.5 Solve problems that involve the volume of 3-D objects and composite 3-D objects in a variety of contexts. 2.6 Solve a problem that involves the capacity of containers. 2.7 Write a given volume measurement expressed in one SI unit cubed in another SI unit cubed. 2.8 Write a given volume measurement expressed in one imperial unit cubed in another imperial unit cubed. 2.9 Determine the volume of prisms, cones, cylinders, pyramids, spheres and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, calipers and micrometers. 2.10 Determine the capacity of prisms, cones, pyramids, spheres and cylinders, using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons and displacement. 2.11 Describe the relationship between the volumes of: • cones and cylinders with the same base and height • pyramids and prisms with the same base and height. 2.12 Illustrate, using examples, the effect of dimensional changes on volume. 2.13 Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container. 2.14 Solve a contextual problem that involves the volume of a 3-D object and requires the manipulation of formulas.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve two and three right triangles. [CN, PS, T, V]	1.1 Identify all of the right triangles in a given illustration for a context. 1.2 Determine if a solution to a problem that involves two or three right triangles is reasonable. 1.3 Sketch a representation of a given description of a problem in a 2-D or 3-D context. 1.4 Solve a contextual problem that involves angles of elevation or angles of depression. 1.5 Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.
2. Solve problems that involve scale. [PS, R, T, V]	2.1 Describe contexts in which a scale representation is used. 2.2 Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model. 2.3 Construct a model of a 3-D object, given the scale. 2.4 Draw, with and without technology, a scale diagram of a given object. 2.5 Solve a contextual problem that involves scale.
3. Model and draw 3-D objects and their views. [CN, R, V]	3.1 Draw a 2-D representation of a given 3-D object. 3.2 Draw, using isometric dot paper, a given 3-D object. 3.3 Draw to scale top, front and side views of a given 3-D object. 3.4 Construct a model of a 3-D object, given the top, front and side views. 3.5 Draw a 3-D object, given the top, front and side views. 3.6 Determine if given views of a 3-D object represent a given object, and explain the reasoning. 3.7 Identify the point of perspective of a given one-point perspective drawing of a 3-D object. 3.8 Draw a one-point perspective view of a given 3-D object.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry (continued)	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Draw and describe exploded views, component parts and scale diagrams of simple 3-D objects. [CN, V]	*(It is intended that the simple 3-D objects come from contexts such as flat-packed furniture or sewing patterns.)* 4.1 Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object. 4.2 Sketch an exploded view of a 3-D object to represent the components. 4.3 Draw to scale the components of a 3-D object. 4.4 Sketch a 2-D representation of a 3-D object, given its exploded view.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	*(It is intended that this outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares and Kakuro.)* 1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. 1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. 1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.
2. Solve problems that involve personal budgets. [CN, PS, R, T]	2.1 Identify income and expenses that should be included in a personal budget. 2.2 Explain considerations that must be made when developing a budget; e.g., prioritizing, recurring and unexpected expenses. 2.3 Create a personal budget based on given income and expense data. 2.4 Collect income and expense data, and create a budget. 2.5 Modify a budget to achieve a set of personal goals. 2.6 Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number (continued)	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Demonstrate an understanding of compound interest. [CN, ME, PS, T]	3.1 Solve a problem that involves simple interest, given three of the four values in the formula $I=Prt$. 3.2 Compare simple and compound interest, and explain their relationship. 3.3 Solve, using a formula, a contextual problem that involves compound interest. 3.4 Explain, using examples, the effect of different compounding periods on calculations of compound interest. 3.5 Estimate, using the Rule of 72, the time required for a given investment to double in value.
4. Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]	4.1 Describe the type of banking services available from various financial institutions, such as online services. 4.2 Describe the types of accounts available at various financial institutions. 4.3 Identify the type of account that best meets the needs for a given set of criteria. 4.4 Identify and explain various automated teller machine (ATM) service charges. 4.5 Describe the advantages and disadvantages of online banking. 4.6 Describe the advantages and disadvantages of debit card purchases. 4.7 Describe ways that ensure the security of personal and financial information; e.g., passwords, encryption, protection of personal identification number (PIN) and other personal identity information.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number (continued)	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
5. Demonstrate an understanding of credit options, including: • credit cards • loans. [CN, ME, PS, R]	5.1 Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, overdraft. 5.2 Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans and sales promotions, and explain the reasoning. 5.3 Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases. 5.4 Compare credit card options from various companies and financial institutions. 5.5 Solve a contextual problem that involves credit cards or loans. 5.6 Solve a contextual problem that involves credit linked to sales promotions.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra	**General Outcome:** Develop algebraic reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • finance charges. [CN, PS, R]	1.1 Solve a contextual problem involving the application of a formula that does not require manipulation. 1.2 Solve a contextual problem involving the application of a formula that requires manipulation. 1.3 Explain and verify why different forms of the same formula are equivalent. 1.4 Describe, using examples, how a given formula is used in a trade or an occupation. 1.5 Create and solve a contextual problem that involves a formula. 1.6 Identify and correct errors in a solution to a problem that involves a formula.
2. Demonstrate an understanding of slope: • as rise over run • as rate of change • by solving problems. [C, CN, PS, V]	2.1 Describe contexts that involve slope; e.g., ramps, roofs, road grade, flow rates within a tube, skateboard parks, ski hills. 2.2 Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications. 2.3 Describe the conditions under which a slope will be either 0 or undefined. 2.4 Explain, using examples and illustrations, slope as rise over run. 2.5 Verify that the slope of an object, such as a ramp or a roof, is constant. 2.6 Explain, using illustrations, the relationship between slope and angle of elevation; e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4^{o}. 2.7 Explain the implications, such as safety and functionality, of different slopes in a given context. 2.8 Explain, using examples and illustrations, slope as rate of change. 2.9 Solve a contextual problem that involves slope or rate of change.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra (continued)	**General Outcome:** Develop algebraic reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]	3.1 Explain the process of unit analysis used to solve a problem (e.g., given km/h and time in hours, determine how many km; given revolutions per minute, determine the number of seconds per revolution). 3.2 Solve a problem, using unit analysis. 3.3 Explain, using an example, how unit analysis and proportional reasoning are related; e.g., to change km/h to km/min, multiply by 1h/60min because hours and minutes are proportional (constant relationship). 3.4 Solve a problem within and between systems, using proportions or tables; e.g., km to m or km/h to ft/sec.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Statistics	**General Outcome:** Develop statistical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve creating and interpreting graphs, including: • bar graphs • histograms • line graphs • circle graphs. [C, CN, PS, R, T, V]	1.1 Determine the possible graphs that can be used to represent a given data set, and explain the advantages and disadvantages of each. 1.2 Create, with and without technology, a graph to represent a given data set. 1.3 Describe the trends in the graph of a given data set. 1.4 Interpolate and extrapolate values from a given graph. 1.5 Explain, using examples, how the same graph can be used to justify more than one conclusion. 1.6 Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view. 1.7 Solve a contextual problem that involves the interpretation of a graph.

APPRENTICESHIP AND WORKPLACE MATHEMATICS GRADE 12

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement	**General Outcome:** Develop spatial sense through direct and indirect measurement.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of the limitations of measuring instruments, including: • precision • accuracy • uncertainty • tolerance and solve problems. [C, PS, R, T, V]	1.1 Explain why, in a given context, a certain degree of precision is required. 1.2 Explain why, in a given context, a certain degree of accuracy is required. 1.3 Explain, using examples, the difference between precision and accuracy. 1.4 Compare the degree of accuracy of two given instruments used to measure the same attribute. 1.5 Relate the degree of accuracy to the uncertainty of a given measure. 1.6 Analyze precision and accuracy in a contextual problem. 1.7 Calculate maximum and minimum values, using a given degree of tolerance in context. 1.8 Describe, using examples, the limitations of measuring instruments used in a specific trade or industry; e.g., tape measure versus Vernier caliper. 1.9 Solve a problem that involves precision, accuracy or tolerance.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]	1.1 Identify and describe the use of the sine law and cosine law in construction, industrial, commercial and artistic applications. 1.2 Solve a problem, using the sine law or cosine law, when a diagram is given.
2. Solve problems that involve: • triangles • quadrilaterals • regular polygons. [C, CN, PS, V]	2.1 Describe and illustrate properties of triangles, including isosceles and equilateral. 2.2 Describe and illustrate properties of quadrilaterals in terms of angle measures, side lengths, diagonal lengths and angles of intersection. 2.3 Describe and illustrate properties of regular polygons. 2.4 Explain, using examples, why a given property does or does not apply to certain polygons. 2.5 Identify and explain an application of the properties of polygons in construction, industrial, commercial, domestic and artistic contexts. 2.6 Solve a contextual problem that involves the application of the properties of polygons.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry (continued)	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including: • translations • rotations • reflections • dilations. [C, CN, R, T, V]	3.1 Identify a single transformation that was performed, given the original 2-D shape or 3-D object and its image. 3.2 Draw the image of a 2-D shape that results from a given single transformation. 3.3 Draw the image of a 2-D shape that results from a given combination of successive transformations. 3.4 Create, analyze and describe designs, using translations, rotations and reflections in all four quadrants of a coordinate grid. 3.5 Identify and describe applications of transformations in construction, industrial, commercial, domestic and artistic contexts. 3.6 Explain the relationship between reflections and lines or planes of symmetry. 3.7 Determine and explain whether a given image is a dilation of another given shape, using the concept of similarity. 3.8 Draw, with or without technology, a dilation image for a given 2-D shape or 3-D object, and explain how the original 2-D shape or 3-D object and its image are proportional. 3.9 Solve a contextual problem that involves transformations.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]	*(It is intended that this outcome be integrated throughout the course by using puzzles and games such as Sudoku, Mastermind, Nim and logic puzzles.)* 1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. 1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. 1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.
2. Solve problems that involve the acquisition of a vehicle by: • buying • leasing • leasing to buy. [C, CN, PS, R, T]	2.1 Describe and explain various options for buying, leasing and leasing to buy a vehicle. 2.2 Solve, with or without technology, problems that involve the purchase, lease or lease to purchase of a vehicle. 2.3 Justify a decision related to buying, leasing or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage and insurance.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Number (continued)	**General Outcome:** Develop number sense and critical thinking skills.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Critique the viability of small business options by considering: • expenses • sales • profit or loss. [C, CN, R]	3.1 Identify expenses in operating a small business, such as a hot dog stand. 3.2 Identify feasible small business options for a given community. 3.3 Generate options that might improve the profitability of a small business. 3.4 Determine the break-even point for a small business. 3.5 Explain factors, such as seasonal variations and hours of operation, that might impact the profitability of a small business.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra	**General Outcome:** Develop algebraic reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of linear relations by: • recognizing patterns and trends • graphing • creating tables of values • writing equations • interpolating and extrapolating • solving problems. [CN, PS, R, T, V]	1.1 Identify and describe the characteristics of a linear relation represented in a graph, table of values, number pattern or equation. 1.2 Sort a set of graphs, tables of values, number patterns and/or equations into linear and nonlinear relations. 1.3 Write an equation for a given context, including direct or partial variation. 1.4 Create a table of values for a given equation of a linear relation. 1.5 Sketch the graph for a given table of values. 1.6 Explain why the points should or should not be connected on the graph for a context. 1.7 Create, with or without technology, a graph to represent a data set, including scatterplots. 1.8 Describe the trends in the graph of a data set, including scatterplots. 1.9 Sort a set of scatterplots according to the trends represented (linear, nonlinear or no trend). 1.10 Solve a contextual problem that requires interpolation or extrapolation of information. 1.11 Relate slope and rate of change to linear relations. 1.12 Match given contexts with their corresponding graphs, and explain the reasoning. 1.13 Solve a contextual problem that involves the application of a formula for a linear relation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Statistics	**General Outcome:** Develop statistical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve measures of central tendency, including: • mean • median • mode • weighted mean • trimmed mean. [C, CN, PS, R]	1.1 Explain, using examples, the advantages and disadvantages of each measure of central tendency. 1.2 Determine the mean, median and mode for a set of data. 1.3 Identify and correct errors in a calculation of a measure of central tendency. 1.4 Identify the outlier(s) in a set of data. 1.5 Explain the effect of outliers on mean, median and mode. 1.6 Calculate the trimmed mean for a set of data, and justify the removal of the outliers. 1.7 Explain, using examples such as course marks, why some data in a set would be given a greater weighting in determining the mean. 1.8 Calculate the mean of a set of numbers after allowing the data to have different weightings (weighted mean). 1.9 Explain, using examples from print and other media, how measures of central tendency and outliers are used to provide different interpretations of data. 1.10 Solve a contextual problem that involves measures of central tendency.
2. Analyze and describe percentiles. [C, CN, PS, R]	2.1 Explain, using examples, percentile ranks in a context. 2.2 Explain decisions based on a given percentile rank. 2.3 Explain, using examples, the difference between percent and percentile rank. 2.4 Explain the relationship between median and percentile. 2.5 Solve a contextual problem that involves percentiles.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Probability	**General Outcome:** Develop critical thinking skills related to uncertainty.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Analyze and interpret problems that involve probability. [C, CN, PS, R]	1.1 Describe and explain the applications of probability; e.g., medication, warranties, insurance, lotteries, weather prediction, 100-year flood, failure of a design, failure of a product, vehicle recalls, approximation of area. 1.2 Calculate the probability of an event based on a data set; e.g., determine the probability of a randomly chosen light bulb being defective. 1.3 Express a given probability as a fraction, decimal and percent and in a statement. 1.4 Explain the difference between odds and probability. 1.5 Determine the probability of an event, given the odds for or against. 1.6 Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results and subjective judgements. 1.7 Solve a contextual problem that involves a given probability.

FOUNDATIONS OF MATHEMATICS AND PRE-CALCULUS GRADE 10

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement	**General Outcome:** Develop spatial sense and proportional reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve linear measurement, using: • SI and imperial units of measure • estimation strategies • measurement strategies. [ME, PS, V]	1.1 Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard and mile, and explain the choices. 1.2 Compare SI and imperial units, using referents. 1.3 Estimate a linear measure, using a referent, and explain the process used. 1.4 Justify the choice of units used for determining a measurement in a problem-solving context. 1.5 Solve problems that involve linear measure, using instruments such as rulers, calipers or tape measures. 1.6 Describe and explain a personal strategy used to determine a linear measurement; e.g., circumference of a bottle, length of a curve, perimeter of the base of an irregular 3-D object.
2. Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]	2.1 Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems. 2.2 Solve a problem that involves the conversion of units within or between SI and imperial systems. 2.3 Verify, using unit analysis, a conversion within or between SI and imperial systems, and explain the conversion. 2.4 Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement (continued)	**General Outcome:** Develop spatial sense and proportional reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including: • right cones • right cylinders • right prisms • right pyramids • spheres. [CN, PS, R, V]	3.1 Sketch a diagram to represent a problem that involves surface area or volume. 3.2 Determine the surface area of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labelled diagram. 3.3 Determine the volume of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labelled diagram. 3.4 Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid or sphere, given the object's surface area or volume and the remaining dimensions. 3.5 Solve a problem that involves surface area or volume, given a diagram of a composite 3-D object. 3.6 Describe the relationship between the volumes of: • right cones and right cylinders with the same base and height • right pyramids and right prisms with the same base and height.
4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]	4.1 Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios. 4.2 Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle. 4.3 Solve right triangles, with or without technology. 4.4 Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean theorem. 4.5 Solve a problem that involves indirect and direct measurement, using the trigonometric ratios, the Pythagorean theorem and measurement instruments such as a clinometer or metre stick.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of factors of whole numbers by determining the: • prime factors • greatest common factor • least common multiple • square root • cube root. [CN, ME, R]	1.1 Determine the prime factors of a whole number. 1.2 Explain why the numbers 0 and 1 have no prime factors. 1.3 Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process. 1.4 Determine, concretely, whether a given whole number is a perfect square, a perfect cube or neither. 1.5 Determine, using a variety of strategies, the square root of a perfect square, and explain the process. 1.6 Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process. 1.7 Solve problems that involve prime factors, greatest common factors, least common multiples, square roots or cube roots.
2. Demonstrate an understanding of irrational numbers by: • representing, identifying and simplifying irrational numbers • ordering irrational numbers. [CN, ME, R, V]	2.1 Sort a set of numbers into rational and irrational numbers. 2.2 Determine an approximate value of a given irrational number. 2.3 Approximate the locations of irrational numbers on a number line, using a variety of strategies, and explain the reasoning. 2.4 Order a set of irrational numbers on a number line. 2.5 Express a radical as a mixed radical in simplest form (limited to numerical radicands). 2.6 Express a mixed radical as an entire radical (limited to numerical radicands). 2.7 Explain, using examples, the meaning of the index of a radical. 2.8 Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number (continued)	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]	3.1 Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$. 3.2 Explain, using patterns, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, $n > 0$. 3.3 Apply the exponent laws: • $(a^m)(a^n) = a^{m+n}$ • $a^m \div a^n = a^{m-n}$, $a \neq 0$ • $(a^m)^n = a^{mn}$ • $(ab)^m = a^m b^m$ • $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$ to expressions with rational and variable bases and integral and rational exponents, and explain the reasoning. 3.4 Express powers with rational exponents as radicals and vice versa. 3.5 Solve a problem that involves exponent laws or radicals. 3.6 Identify and correct errors in a simplification of an expression that involves powers.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number (continued)	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V]	*(It is intended that the emphasis of this outcome be on binomial by binomial multiplication, with extension to polynomial by polynomial to establish a general pattern for multiplication.)* 4.1 Model the multiplication of two given binomials, concretely or pictorially, and record the process symbolically. 4.2 Relate the multiplication of two binomial expressions to an area model. 4.3 Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers. 4.4 Verify a polynomial product by substituting numbers for the variables. 4.5 Multiply two polynomials symbolically, and combine like terms in the product. 4.6 Generalize and explain a strategy for multiplication of polynomials. 4.7 Identify and explain errors in a solution for a polynomial multiplication.
5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]	5.1 Determine the common factors in the terms of a polynomial, and express the polynomial in factored form. 5.2 Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically. 5.3 Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where $b = 0$. 5.4 Identify and explain errors in a polynomial factorization. 5.5 Factor a polynomial, and verify by multiplying the factors. 5.6 Explain, using examples, the relationship between multiplication and factoring of polynomials. 5.7 Generalize and explain strategies used to factor a trinomial. 5.8 Express a polynomial as a product of its factors.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Interpret and explain the relationships among data, graphs and situations. [C, CN, R, T, V]	1.1 Graph, with or without technology, a set of data, and determine the restrictions on the domain and range. 1.2 Explain why data points should or should not be connected on the graph for a situation. 1.3 Describe a possible situation for a given graph. 1.4 Sketch a possible graph for a given situation. 1.5 Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs or a table of values.
2. Demonstrate an understanding of relations and functions. [C, R, V]	2.1 Explain, using examples, why some relations are not functions but all functions are relations. 2.2 Determine if a set of ordered pairs represents a function. 2.3 Sort a set of graphs as functions or non-functions. 2.4 Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.
3. Demonstrate an understanding of slope with respect to: • rise and run • line segments and lines • rate of change • parallel lines • perpendicular lines. [PS, R, V]	3.1 Determine the slope of a line segment by measuring or calculating the rise and run. 3.2 Classify lines in a given set as having positive or negative slopes. 3.3 Explain the meaning of the slope of a horizontal or vertical line. 3.4 Explain why the slope of a line can be determined by using any two points on that line. 3.5 Explain, using examples, slope as a rate of change. 3.6 Draw a line, given its slope and a point on the line. 3.7 Determine another point on a line, given the slope and a point on the line. 3.8 Generalize and apply a rule for determining whether two lines are parallel or perpendicular. 3.9 Solve a contextual problem involving slope.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Describe and represent linear relations, using: • words • ordered pairs • tables of values • graphs • equations. [C, CN, R, V]	4.1 Identify independent and dependent variables in a given context. 4.2 Determine whether a situation represents a linear relation, and explain why or why not. 4.3 Determine whether a graph represents a linear relation, and explain why or why not. 4.4 Determine whether a table of values or a set of ordered pairs represents a linear relation, and explain why or why not. 4.5 Draw a graph from a set of ordered pairs within a given situation, and determine whether the relationship between the variables is linear. 4.6 Determine whether an equation represents a linear relation, and explain why or why not. 4.7 Match corresponding representations of linear relations.
5. Determine the characteristics of the graphs of linear relations, including the: • intercepts • slope • domain • range. [CN, PS, R, V]	5.1 Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs. 5.2 Determine the slope of the graph of a linear relation. 5.3 Determine the domain and range of the graph of a linear relation. 5.4 Sketch a linear relation that has one intercept, two intercepts or an infinite number of intercepts. 5.5 Identify the graph that corresponds to a given slope and y-intercept. 5.6 Identify the slope and y-intercept that correspond to a given graph. 5.7 Solve a contextual problem that involves intercepts, slope, domain or range of a linear relation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
6. Relate linear relations expressed in: • slope–intercept form ($y = mx + b$) • general form ($Ax + By + C = 0$) • slope–point form ($y - y_1 = m(x - x_1)$) to their graphs. [CN, R, T, V]	6.1 Express a linear relation in different forms, and compare the graphs. 6.2 Rewrite a linear relation in either slope–intercept or general form. 6.3 Generalize and explain strategies for graphing a linear relation in slope–intercept, general or slope–point form. 6.4 Graph, with and without technology, a linear relation given in slope–intercept, general or slope–point form, and explain the strategy used to create the graph. 6.5 Identify equivalent linear relations from a set of linear relations. 6.6 Match a set of linear relations to their graphs.
7. Determine the equation of a linear relation, given: • a graph • a point and the slope • two points • a point and the equation of a parallel or perpendicular line to solve problems. [CN, PS, R, V]	7.1 Determine the slope and y-intercept of a given linear relation from its graph, and write the equation in the form $y = mx + b$. 7.2 Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the reasoning. 7.3 Write the equation of a linear relation, given the coordinates of two points on the line, and explain the reasoning. 7.4 Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning. 7.5 Graph linear data generated from a context, and write the equation of the resulting line. 7.6 Solve a problem, using the equation of a linear relation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
8. Represent a linear function, using function notation. [CN, ME, V]	8.1 Express the equation of a linear function in two variables, using function notation. 8.2 Express an equation given in function notation as a linear function in two variables. 8.3 Determine the related range value, given a domain value for a linear function; e.g., if $f(x) = 3x - 2$, determine $f(-1)$. 8.4 Determine the related domain value, given a range value for a linear function; e.g., if $g(t) = 7 + t$, determine t so that $g(t) = 15$. 8.5 Sketch the graph of a linear function expressed in function notation.
9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]	9.1 Model a situation, using a system of linear equations. 9.2 Relate a system of linear equations to the context of a problem. 9.3 Determine and verify the solution of a system of linear equations graphically, with and without technology. 9.4 Explain the meaning of the point of intersection of a system of linear equations. 9.5 Determine and verify the solution of a system of linear equations algebraically. 9.6 Explain, using examples, why a system of equations may have no solution, one solution or an infinite number of solutions. 9.7 Explain a strategy to solve a system of linear equations. 9.8 Solve a problem that involves a system of linear equations.

FOUNDATIONS OF MATHEMATICS
GRADE 11

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement	**General Outcome:** Develop spatial sense and proportional reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve the application of rates. [CN, PS, R]	1.1 Interpret rates in a given context, such as the arts, commerce, the environment, medicine or recreation. 1.2 Solve a rate problem that requires the isolation of a variable. 1.3 Determine and compare rates and unit rates. 1.4 Make and justify a decision, using rates. 1.5 Represent a given rate pictorially. 1.6 Draw a graph to represent a rate. 1.7 Explain, using examples, the relationship between the slope of a graph and a rate. 1.8 Describe a context for a given rate or unit rate. 1.9 Identify and explain factors that influence a rate in a given context. 1.10 Solve a contextual problem that involves rates or unit rates.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Measurement (continued)	**General Outcome:** Develop spatial sense and proportional reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, V]	2.1 Explain, using examples, how scale diagrams are used to model a 2-D shape or a 3-D object. 2.2 Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape or a 3-D object and its representation. 2.3 Determine, using proportional reasoning, an unknown dimension of a 2-D shape or a 3-D object, given a scale diagram or a model. 2.4 Draw, with or without technology, a scale diagram of a given 2-D shape according to a specified scale factor (enlargement or reduction). 2.5 Solve a contextual problem that involves scale diagrams.
3. Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects. [C, CN, PS, R, V]	3.1 Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result. 3.2 Determine the surface area and volume of a 3-D object, given the scale diagram, and justify the reasonableness of the result. 3.3 Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape. 3.4 Explain, using examples, the effect of a change in the scale factor on the surface area of a 3-D object. 3.5 Explain, using examples, the effect of a change in the scale factor on the volume of a 3-D object. 3.6 Explain, using examples, the relationships among scale factor, area of a 2-D shape, surface area of a 3-D object and volume of a 3-D object. 3.7 Solve a spatial problem that requires the manipulation of formulas. 3.8 Solve a contextual problem that involves the relationships among scale factors, areas and volumes.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Geometry	**General Outcome:** Develop spatial sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Derive proofs that involve the properties of angles and triangles. [CN, R, V]	*(It is intended that deductive reasoning be limited to direct proof.)* 1.1 Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology. 1.2 Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle. 1.3 Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon, with or without technology. 1.4 Identify and correct errors in a given proof of a property involving angles. 1.5 Verify, with examples, that if lines are not parallel the angle properties do not apply.
2. Solve problems that involve the properties of angles and triangles. [CN, PS, V]	2.1 Determine the measures of angles in a diagram that involves parallel lines, angles and triangles, and justify the reasoning. 2.2 Identify and correct errors in a given solution to a problem that involves the measures of angles. 2.3 Solve a contextual problem that involves angles or triangles. 2.4 Construct parallel lines, using only a compass or a protractor, and explain the strategy used. 2.5 Determine if lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.
3. Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R]	3.1 Draw a diagram to represent a problem that involves the cosine law or sine law. 3.2 Explain the steps in a given proof of the sine law or cosine law. 3.3 Solve a problem involving the cosine law that requires the manipulation of a formula. 3.4 Explain, concretely, pictorially or symbolically, whether zero, one or two triangles exist, given two sides and a non-included angle. 3.5 Solve a problem involving the sine law that requires the manipulation of a formula. 3.6 Solve a contextual problem that involves the cosine law or the sine law.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Logical Reasoning	**General Outcome:** Develop logical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems. [C, CN, PS, R]	1.1 Make conjectures by observing patterns and identifying properties, and justify the reasoning. 1.2 Explain why inductive reasoning may lead to a false conjecture. 1.3 Compare, using examples, inductive and deductive reasoning. 1.4 Provide and explain a counterexample to disprove a given conjecture. 1.5 Prove algebraic and number relationships, such as divisibility rules, number properties, mental mathematics strategies or algebraic number tricks. 1.6 Prove a conjecture, using deductive reasoning (not limited to two column proofs). 1.7 Determine if a given argument is valid, and justify the reasoning. 1.8 Identify errors in a given proof; e.g., a proof that ends with $2 = 1$. 1.9 Solve a contextual problem involving inductive or deductive reasoning.
2. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, V]	*(It is intended that this outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction and similar puzzles and games.)* 2.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. 2.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. 2.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Statistics	**General Outcome:** Develop statistical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of normal distribution, including: • standard deviation • *z*-scores. [CN, PS, T, V]	1.1 Explain, using examples, the meaning of standard deviation. 1.2 Calculate, using technology, the population standard deviation of a data set. 1.3 Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry and area under the curve. 1.4 Determine if a data set approximates a normal distribution, and explain the reasoning. 1.5 Compare the properties of two or more normally distributed data sets. 1.6 Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in situations such as warranties, insurance or opinion polls. 1.7 Solve a contextual problem that involves the interpretation of standard deviation. 1.8 Determine, with or without technology, and explain the *z*-score for a given value in a normally distributed data set. 1.9 Solve a contextual problem that involves normal distribution.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Statistics (continued)	**General Outcome:** Develop statistical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Interpret statistical data, using: • confidence intervals • confidence levels • margin of error. [C, CN, R]	*(It is intended that the focus of this outcome be on interpretation of data rather than on statistical calculations.)* 2.1 Explain, using examples, how confidence levels, margin of error and confidence intervals may vary depending on the size of the random sample. 2.2 Explain, using examples, the significance of a confidence interval, margin of error or confidence level. 2.3 Make inferences about a population from sample data, using given confidence intervals, and explain the reasoning. 2.4 Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position. 2.5 Interpret and explain confidence intervals and margin of error, using examples found in print or electronic media. 2.6 Support a position by analyzing statistical data presented in the media.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Model and solve problems that involve systems of linear inequalities in two variables. [CN, PS, T, V]	1.1 Model a problem, using a system of linear inequalities in two variables. 1.2 Graph the boundary line between two half planes for each inequality in a system of linear inequalities, and justify the choice of solid or broken lines. 1.3 Determine and explain the solution region that satisfies a linear inequality, using a test point when given a boundary line. 1.4 Determine, graphically, the solution region for a system of linear inequalities, and verify the solution. 1.5 Explain, using examples, the significance of the shaded region in the graphical solution of a system of linear inequalities. 1.6 Solve an optimization problem, using linear programming.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex • intercepts • domain and range • axis of symmetry. [CN, PS, T, V]	*(It is intended that completion of the square not be required.)* 2.1 Determine, with or without technology, the intercepts of the graph of a quadratic function. 2.2 Determine, by factoring, the roots of a quadratic equation, and verify by substitution. 2.3 Determine, using the quadratic formula, the roots of a quadratic equation. 2.4 Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the *x*-intercepts of the graph of the function. 2.5 Explain, using examples, why the graph of a quadratic function may have zero, one or two *x*-intercepts. 2.6 Express a quadratic equation in factored form, using the zeros of a corresponding function or the *x*-intercepts of its graph. 2.7 Determine, with or without technology, the coordinates of the vertex of the graph of a quadratic function. 2.8 Determine the equation of the axis of symmetry of the graph of a quadratic function, given the *x*-intercepts of the graph. 2.9 Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine if the *y*-coordinate of the vertex is a maximum or a minimum. 2.10 Determine the domain and range of a quadratic function. 2.11 Sketch the graph of a quadratic function. 2.12 Solve a contextual problem that involves the characteristics of a quadratic function.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Mathematics Research Project	**General Outcome:** Develop an appreciation of the role of mathematics in society.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Research and give a presentation on a historical event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	1.1 Collect primary or secondary data (statistical or informational) related to the topic. 1.2 Assess the accuracy, reliability and relevance of the primary or secondary data collected by: • identifying examples of bias and points of view • identifying and describing the data collection methods • determining if the data is relevant • determining if the data is consistent with information obtained from other sources on the same topic. 1.3 Interpret data, using statistical methods if applicable. 1.4 Identify controversial issues, if any, and present multiple sides of the issues with supporting data. 1.5 Organize and present the research project, with or without technology.

FOUNDATIONS OF MATHEMATICS
GRADE 12

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Financial Mathematics	**General Outcome:** Develop number sense in financial applications.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V]	1.1 Explain the advantages and disadvantages of compound interest and simple interest. 1.2 Identify situations that involve compound interest. 1.3 Graph and compare, in a given situation, the total interest paid or earned for different compounding periods. 1.4 Determine, given the principal, interest rate and number of compounding periods, the total interest of a loan. 1.5 Graph and describe the effects of changing the value of one of the variables in a situation that involves compound interest. 1.6 Determine, using technology, the total cost of a loan under a variety of conditions; e.g., different amortization periods, interest rates, compounding periods and terms. 1.7 Compare and explain, using technology, different credit options that involve compound interest, including bank and store credit cards and special promotions. 1.8 Solve a contextual problem that involves compound interest.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Financial Mathematics (continued)	**General Outcome:** Develop number sense in financial applications.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Analyze costs and benefits of renting, leasing and buying. [CN, PS, R, T]	2.1 Identify and describe examples of assets that appreciate or depreciate. 2.2 Compare, using examples, renting, leasing and buying. 2.3 Justify, for a specific set of circumstances, if renting, buying or leasing would be advantageous. 2.4 Solve a problem involving renting, leasing or buying that requires the manipulation of a formula. 2.5 Solve, using technology, a contextual problem that involves cost-and-benefit analysis.
3. Analyze an investment portfolio in terms of: • interest rate • rate of return • total return. [ME, PS, R, T]	3.1 Determine and compare the strengths and weaknesses of two or more portfolios. 3.2 Determine, using technology, the total value of an investment when there are regular contributions to the principal. 3.3 Graph and compare the total value of an investment with and without regular contributions. 3.4 Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule. 3.5 Determine, using technology, possible investment strategies to achieve a financial goal. 3.6 Explain the advantages and disadvantages of long-term and short-term investment options. 3.7 Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term. 3.8 Solve an investment problem.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Logical Reasoning	**General Outcome:** Develop logical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]	*(It is intended that this outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro and cribbage.)* 1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches. 1.2 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. 1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Logical Reasoning (continued)	**General Outcome:** Develop logical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Solve problems that involve the application of set theory. [CN, PS, R, V]	2.1 Provide examples of the empty set, disjoint sets, subsets and universal sets in context, and explain the reasoning. 2.2 Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning. 2.3 Explain what a specified region in a Venn diagram represents, using connecting words (and, or, not) or set notation. 2.4 Determine the elements in the complement, the intersection or the union of two sets. 2.5 Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games and puzzles. 2.6 Identify and correct errors in a given solution to a problem that involves sets. 2.7 Solve a contextual problem that involves sets, and record the solution, using set notation.
3. Solve problems that involve conditional statements. [C, CN, PS, R]	3.1 Analyze an “if-then” statement, make a conclusion, and explain the reasoning. 3.2 Make and justify a decision, using “what if?” questions, in contexts such as probability, finance, sports, games or puzzles, with or without technology. 3.3 Determine the converse, inverse and contrapositive of an “if-then” statement; determine its veracity; and, if it is false, provide a counterexample. 3.4 Demonstrate, using examples, that the veracity of any statement does not imply the veracity of its converse or inverse. 3.5 Demonstrate, using examples, that the veracity of any statement does imply the veracity of its contrapositive. 3.6 Identify and describe contexts in which a biconditional statement can be justified. 3.7 Analyze and summarize, using a graphic organizer such as a truth table or Venn diagram, the possible results of given logical arguments that involve biconditional, converse, inverse or contrapositive statements.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Probability	**General Outcome:** Develop critical thinking skills related to uncertainty.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Interpret and assess the validity of odds and probability statements. [C, CN, ME]	1.1 Provide examples of statements of probability and odds found in fields such as media, biology, sports, medicine, sociology and psychology. 1.2 Explain, using examples, the relationship between odds (part-part) and probability (part-whole). 1.3 Express odds as a probability and vice versa. 1.4 Determine the probability of, or the odds for and against, an outcome in a situation. 1.5 Explain, using examples, how decisions may be based on probability or odds and on subjective judgments. 1.6 Solve a contextual problem that involves odds or probability.
2. Solve problems that involve the probability of mutually exclusive and non–mutually exclusive events. [CN, PS, R, V]	2.1 Classify events as mutually exclusive or non–mutually exclusive, and explain the reasoning. 2.2 Determine if two events are complementary, and explain the reasoning. 2.3 Represent, using set notation or graphic organizers, mutually exclusive (including complementary) and non–mutually exclusive events. 2.4 Solve a contextual problem that involves the probability of mutually exclusive or non–mutually exclusive events. 2.5 Solve a contextual problem that involves the probability of complementary events. 2.6 Create and solve a problem that involves mutually exclusive or non–mutually exclusive events.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Probability (continued)	**General Outcome:** Develop critical thinking skills related to uncertainty.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Solve problems that involve the probability of two events. [CN, PS, R]	3.1 Compare, using examples, dependent and independent events. 3.2 Determine the probability of an event, given the occurrence of a previous event. 3.3 Determine the probability of two dependent or two independent events. 3.4 Create and solve a contextual problem that involves determining the probability of dependent or independent events.
4. Solve problems that involve the fundamental counting principle. [PS, R, V]	4.1 Represent and solve counting problems, using a graphic organizer. 4.2 Generalize the fundamental counting principle, using inductive reasoning. 4.3 Identify and explain assumptions made in solving a counting problem. 4.4 Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.
5. Solve problems that involve permutations. [ME, PS, R, T, V]	*(It is intended that circular permutations not be included.)* 5.1 Represent the number of arrangements of n elements taken n at a time, using factorial notation. 5.2 Determine, with or without technology, the value of a factorial. 5.3 Simplify a numeric or algebraic fraction containing factorials in both the numerator and denominator. 5.4 Solve an equation that involves factorials. 5.5 Determine the number of permutations of n elements taken r at a time. 5.6 Determine the number of permutations of n elements taken n at a time where some elements are not distinct. 5.7 Explain, using examples, the effect on the total number of permutations of n elements when two or more elements are identical. 5.8 Generalize strategies for determining the number of permutations of n elements taken r at a time. 5.9 Solve a contextual problem that involves probability and permutations.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Probability (continued)	**General Outcome:** Develop critical thinking skills related to uncertainty.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
6. Solve problems that involve combinations. [ME, PS, R, T, V]	6.1 Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations. 6.2 Determine the number of combinations of *n* elements taken *r* at a time. 6.3 Generalize strategies for determining the number of combinations of *n* elements taken *r* at a time. 6.4 Solve a contextual problem that involves combinations and probability.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Represent data, using polynomial functions (of degree ≤ 3), to solve problems. [C, CN, PS, T, V]	1.1 Describe, orally and in written form, the characteristics of polynomial functions by analyzing their graphs. 1.2 Describe, orally and in written form, the characteristics of polynomial functions by analyzing their equations. 1.3 Match equations in a given set to their corresponding graphs. 1.4 Graph data and determine the polynomial function that best approximates the data. 1.5 Interpret the graph of a polynomial function that models a situation, and explain the reasoning. 1.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of polynomial functions, and explain the reasoning.
2. Represent data, using exponential and logarithmic functions, to solve problems. [C, CN, PS, T, V]	2.1 Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analyzing their graphs. 2.2 Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analyzing their equations. 2.3 Match equations in a given set to their corresponding graphs. 2.4 Graph data and determine the exponential or logarithmic function that best approximates the data. 2.5 Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning. 2.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Represent data, using sinusoidal functions, to solve problems. [C, CN, PS, T, V]	3.1 Describe, orally and in written form, the characteristics of sinusoidal functions by analyzing their graphs. 3.2 Describe, orally and in written form, the characteristics of sinusoidal functions by analyzing their equations. 3.3 Match equations in a given set to their corresponding graphs. 3.4 Graph data and determine the sinusoidal function that best approximates the data. 3.5 Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning. 3.6 Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Mathematics Research Project	**General Outcome:** Develop an appreciation of the role of mathematics in society.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	1.1 Collect primary or secondary data (statistical or informational) related to the topic. 1.2 Assess the accuracy, reliability and relevance of the primary or secondary data collected by: • identifying examples of bias and points of view • identifying and describing the data collection methods • determining if the data is relevant • determining if the data is consistent with information obtained from other sources on the same topic. 1.3 Interpret data, using statistical methods if applicable. 1.4 Identify controversial issues, if any, and present multiple sides of the issues with supporting data. 1.5 Organize and present the research project, with or without technology.

PRE-CALCULUS
GRADE 11

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of the absolute value of real numbers. [R, V]	1.1 Determine the distance of two real numbers of the form $\pm a, a \in R$, from 0 on a number line, and relate this to the absolute value of a ($\lvert a \rvert$). 1.2 Determine the absolute value of a positive or negative real number. 1.3 Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value. 1.4 Determine the absolute value of a numerical expression. 1.5 Compare and order the absolute values of real numbers in a given set.
2. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T]	2.1 Compare and order radical expressions with numerical radicands in a given set. 2.2 Express an entire radical with a numerical radicand as a mixed radical. 2.3 Express a mixed radical with a numerical radicand as an entire radical. 2.4 Perform one or more operations to simplify radical expressions with numerical or variable radicands. 2.5 Rationalize the denominator of a rational expression with monomial or binomial denominators. 2.6 Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression. 2.7 Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = \lvert x \rvert$ and $\sqrt{x^2} \neq \pm x$; e.g., $\sqrt{9} \neq \pm 3$. 2.8 Identify the values of the variable for which a given radical expression is defined. 2.9 Solve a problem that involves radical expressions.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number (continued)	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Solve problems that involve radical equations (limited to square roots). [C, PS, R]	*(It is intended that the equations will have no more than two radicals.)* 3.1 Determine any restrictions on values for the variable in a radical equation. 3.2 Determine the roots of a radical equation algebraically, and explain the process used to solve the equation. 3.3 Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation. 3.4 Explain why some roots determined in solving a radical equation algebraically are extraneous. 3.5 Solve problems by modelling a situation using a radical equation.
4. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [C, ME, R]	4.1 Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers. 4.2 Explain why a given value is non-permissible for a given rational expression. 4.3 Determine the non-permissible values for a rational expression. 4.4 Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression. 4.5 Simplify a rational expression. 4.6 Explain why the non-permissible values of a given rational expression and its simplified form are the same. 4.7 Identify and correct errors in a simplification of a rational expression, and explain the reasoning.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Algebra and Number (continued)	**General Outcome:** Develop algebraic reasoning and number sense.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
5. Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, ME, R]	5.1 Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers. 5.2 Determine the non-permissible values when performing operations on rational expressions. 5.3 Determine, in simplified form, the sum or difference of rational expressions with the same denominator. 5.4 Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors. 5.5 Determine, in simplified form, the product or quotient of rational expressions. 5.6 Simplify an expression that involves two or more operations on rational expressions.
6. Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). [C, PS, R]	*(It is intended that the rational equations be those that can be simplified to linear and quadratic equations.)* 6.1 Determine the non-permissible values for the variable in a rational equation. 6.2 Determine the solution to a rational equation algebraically, and explain the process used to solve the equation. 6.3 Explain why a value obtained in solving a rational equation may not be a solution of the equation. 6.4 Solve problems by modelling a situation using a rational equation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Trigonometry	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of angles in standard position [0° to 360°]. [R, V]	1.1 Sketch an angle in standard position, given the measure of the angle. 1.2 Determine the reference angle for an angle in standard position. 1.3 Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle. 1.4 Illustrate, using examples, that any angle from 90° to 360° is the reflection in the *x*-axis and/or the *y*-axis of its reference angle. 1.5 Determine the quadrant in which a given angle in standard position terminates. 1.6 Draw an angle in standard position given any point $P(x, y)$ on the terminal arm of the angle. 1.7 Illustrate, using examples, that the points $P(x, y)$, $P(-x, y)$, $P(-x, -y)$ and $P(x, -y)$ are points on the terminal sides of angles in standard position that have the same reference angle.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position. [C, ME, PS, R, T, V]	2.1 Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point $P(x, y)$ on the terminal arm of an angle. 2.2 Determine the value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, given any point $P(x, y)$ on the terminal arm of angle θ. 2.3 Determine, without the use of technology, the value of $\sin\theta$, $\cos\theta$ or $\tan\theta$, given any point $P(x, y)$ on the terminal arm of angle θ, where θ = 0º, 90º, 180º, 270º or 360º. 2.4 Determine the sign of a given trigonometric ratio for a given angle, without the use of technology, and explain. 2.5 Solve, for all values of θ, an equation of the form $\sin\theta = a$ or $\cos\theta = a$, where $-1 \le a \le 1$, and an equation of the form $\tan\theta = a$, where a is a real number. 2.6 Determine the exact value of the sine, cosine or tangent of a given angle with a reference angle of 30º, 45º or 60º. 2.7 Describe patterns in and among the values of the sine, cosine and tangent ratios for angles from 0° to 360°. 2.8 Sketch a diagram to represent a problem. 2.9 Solve a contextual problem, using trigonometric ratios.
3. Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	3.1 Sketch a diagram to represent a problem that involves a triangle without a right angle. 3.2 Solve, using primary trigonometric ratios, a triangle that is not a right triangle. 3.3 Explain the steps in a given proof of the sine law or cosine law. 3.4 Sketch a diagram and solve a problem, using the cosine law. 3.5 Sketch a diagram and solve a problem, using the sine law. 3.6 Describe and explain situations in which a problem may have no solution, one solution or two solutions.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Factor polynomial expressions of the form: • $ax^2+bx+c,\ a\neq 0$ • $a^2x^2-b^2y^2,\ a\neq 0,\ b\neq 0$ • $a\left(f\left(x\right)\right)^2+b\left(f\left(x\right)\right)+c,\ a\neq 0$ • $a^2\left(f\left(x\right)\right)^2-b^2\left(g\left(y\right)\right)^2,\ a\neq 0,\ b\neq 0$ where a, b and c are rational numbers. [CN, ME, R]	1.1 Factor a given polynomial expression that requires the identification of common factors. 1.2 Determine whether a given binomial is a factor for a given polynomial expression, and explain why or why not. 1.3 Factor a given polynomial expression of the form: • $ax^2+bx+c,\ a\neq 0$ • $a^2x^2-b^2y^2,\ a\neq 0,\ b\neq 0$. 1.4 Factor a given polynomial expression that has a quadratic pattern, including: • $a\left(f\left(x\right)\right)^2+b\left(f\left(x\right)\right)+c,\ a\neq 0$ • $a^2\left(f\left(x\right)\right)^2-b^2\left(g\left(y\right)\right)^2,\ a\neq 0,\ b\neq 0$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V]	2.1 Create a table of values for $y=\left\|f\left(x\right)\right\|$, given a table of values for $y=f\left(x\right)$. 2.2 Generalize a rule for writing absolute value functions in piecewise notation. 2.3 Sketch the graph of $y=\left\|f\left(x\right)\right\|$; state the intercepts, domain and range; and explain the strategy used. 2.4 Solve an absolute value equation graphically, with or without technology. 2.5 Solve, algebraically, an equation with a single absolute value, and verify the solution. 2.6 Explain why the absolute value equation $\left\|f\left(x\right)\right\|<0$ has no solution. 2.7 Determine and correct errors in a solution to an absolute value equation. 2.8 Solve a problem that involves an absolute value function.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts. [CN, R, T, V]	3.1 Explain why a function given in the form $y = a(x - p)^2 + q$ is a quadratic function. 3.2 Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of a. 3.3 Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of q. 3.4 Compare the graphs of a set of functions of the form $y = (x - p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of p. 3.5 Determine the coordinates of the vertex for a quadratic function of the form $y = a(x - p)^2 + q$, and verify with or without technology. 3.6 Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x - p)^2 + q$. 3.7 Sketch the graph of $y = a(x - p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry and x- and y-intercepts. 3.8 Explain, using examples, how the values of a and q may be used to determine whether a quadratic function has zero, one or two x-intercepts. 3.9 Write a quadratic function in the form $y = a(x - p)^2 + q$ for a given graph or a set of characteristics of a graph.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts and to solve problems. [CN, PS, R, T, V]	4.1 Explain the reasoning for the process of completing the square as shown in a given example. 4.2 Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 + q$ by completing the square. 4.3 Identify, explain and correct errors in an example of completing the square. 4.4 Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used. 4.5 Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$. 4.6 Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function in the form $y = a(x - p)^2 + q$. 4.7 Write a quadratic function that models a given situation, and explain any assumptions made. 4.8 Solve a problem, with or without technology, by analyzing a quadratic function.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
5. Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]	5.1 Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the *x*-intercepts of the graph of the quadratic function. 5.2 Derive the quadratic formula, using deductive reasoning. 5.3 Solve a quadratic equation of the form $ax^2+bx+c=0$ by using strategies such as: • determining square roots • factoring • completing the square • applying the quadratic formula • graphing its corresponding function. 5.4 Select a method for solving a quadratic equation, justify the choice, and verify the solution. 5.5 Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function. 5.6 Identify and correct errors in a solution to a quadratic equation. 5.7 Solve a problem by: • analyzing a quadratic equation • determining and analyzing a quadratic equation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
6. Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. [CN, PS, R, T, V]	*(It is intended that the quadratic equations be limited to those that correspond to quadratic functions.)* 6.1 Model a situation, using a system of linear-quadratic or quadratic-quadratic equations. 6.2 Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a given problem. 6.3 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology. 6.4 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically. 6.5 Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations. 6.6 Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two or an infinite number of solutions. 6.7 Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
7. Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]	7.1 Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality. 7.2 Explain, using examples, when a solid or broken line should be used in the solution for an inequality. 7.3 Sketch, with or without technology, the graph of a linear or quadratic inequality. 7.4 Solve a problem that involves a linear or quadratic inequality.
8. Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]	8.1 Determine the solution of a quadratic inequality in one variable, using strategies such as case analysis, graphing, roots and test points, or sign analysis; and explain the strategy used. 8.2 Represent and solve a problem that involves a quadratic inequality in one variable. 8.3 Interpret the solution to a problem that involves a quadratic inequality in one variable.
9. Analyze arithmetic sequences and series to solve problems. [CN, PS, R, T]	9.1 Identify the assumption(s) made when defining an arithmetic sequence or series. 9.2 Provide and justify an example of an arithmetic sequence. 9.3 Derive a rule for determining the general term of an arithmetic sequence. 9.4 Describe the relationship between arithmetic sequences and linear functions. 9.5 Determine t_1, d, n or t_n in a problem that involves an arithmetic sequence. 9.6 Derive a rule for determining the sum of n terms of an arithmetic series. 9.7 Determine t_1, d, n or S_n in a problem that involves an arithmetic series. 9.8 Solve a problem that involves an arithmetic sequence or series.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
10. Analyze geometric sequences and series to solve problems. [PS, R, T]	10.1 Identify assumptions made when identifying a geometric sequence or series. 10.2 Provide and justify an example of a geometric sequence. 10.3 Derive a rule for determining the general term of a geometric sequence. 10.4 Determine t_1, r, n or t_n in a problem that involves a geometric sequence. 10.5 Derive a rule for determining the sum of n terms of a geometric series. 10.6 Determine t_1, r, n or S_n in a problem that involves a geometric series. 10.7 Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series. 10.8 Explain why a geometric series is convergent or divergent. 10.9 Solve a problem that involves a geometric sequence or series.
11. Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions). [CN, R, T, V]	11.1 Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$. 11.2 Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression. 11.3 Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used. 11.4 Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.

PRE-CALCULUS
GRADE 12

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Trigonometry	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of angles in standard position, expressed in degrees and radians. [CN, ME, R, V]	1.1 Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees. 1.2 Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees. 1.3 Sketch, in standard position, an angle with a measure of 1 radian. 1.4 Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians, where $k \in Q$. 1.5 Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees. 1.6 Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation). 1.7 Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with a given angle in standard position. 1.8 Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with a given angle in standard position. 1.9 Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius *r*, and solve problems based upon that relationship.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
2. Develop and apply the equation of the unit circle. [CN, R, V]	2.1 Derive the equation of the unit circle from the Pythagorean theorem. 2.2 Describe the six trigonometric ratios, using a point P (x, y) that is the intersection of the terminal arm of an angle and the unit circle. 2.3 Generalize the equation of a circle with centre (0, 0) and radius r.
3. Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. [ME, PS, R, T, V]	3.1 Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians. 3.2 Determine, using a unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0º, 30º, 45º, 60º or 90º, or for angles expressed in radians that are multiples of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ or $\frac{\pi}{2}$, and explain the strategy. 3.3 Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio. 3.4 Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position. 3.5 Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position. 3.6 Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain. 3.7 Sketch a diagram to represent a problem that involves trigonometric ratios. 3.8 Solve a problem, using trigonometric ratios.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems. [CN, PS, T, V]	4.1 Sketch, with or without technology, the graph of $y = \sin x$, $y = \cos x$ or $y = \tan x$. 4.2 Determine the characteristics (amplitude, asymptotes, domain, period, range and zeros) of the graph of $y = \sin x$, $y = \cos x$ or $y = \tan x$. 4.3 Determine how varying the value of a affects the graphs of $y = a \sin x$ and $y = a \cos x$. 4.4 Determine how varying the value of d affects the graphs of $y = \sin x + d$ and $y = \cos x + d$. 4.5 Determine how varying the value of c affects the graphs of $y = \sin (x + c)$ and $y = \cos (x + c)$. 4.6 Determine how varying the value of b affects the graphs of $y = \sin bx$ and $y = \cos bx$. 4.7 Sketch, without technology, graphs of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, using transformations, and explain the strategies. 4.8 Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graph of a trigonometric function of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$. 4.9 Determine the values of a, b, c and d for functions of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$ that correspond to a given graph, and write the equation of the function. 4.10 Determine a trigonometric function that models a situation to solve a problem. 4.11 Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation. 4.12 Solve a problem by analyzing the graph of a trigonometric function.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Trigonometry (continued)	**General Outcome:** Develop trigonometric reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
5. Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians. [CN, PS, R, T, V]	5.1 Verify, with or without technology, that a given value is a solution to a trigonometric equation. 5.2 Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible. 5.3 Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain. 5.4 Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions). 5.5 Determine, using technology, the general solution of a given trigonometric equation. 5.6 Identify and correct errors in a solution for a trigonometric equation.
6. Prove trigonometric identities, using: • reciprocal identities • quotient identities • Pythagorean identities • sum or difference identities (restricted to sine, cosine and tangent) • double-angle identities (restricted to sine, cosine and tangent). [R, T, V]	6.1 Explain the difference between a trigonometric identity and a trigonometric equation. 6.2 Verify a trigonometric identity numerically for a given value in either degrees or radians. 6.3 Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid. 6.4 Determine, graphically, the potential validity of a trigonometric identity, using technology. 6.5 Determine the non-permissible values of a trigonometric identity. 6.6 Prove, algebraically, that a trigonometric identity is valid. 6.7 Determine, using the sum, difference and double-angle identities, the exact value of a trigonometric ratio.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of operations on, and compositions of, functions. [CN, R, T, V]	1.1 Sketch the graph of a function that is the sum, difference, product or quotient of two functions, given their graphs. 1.2 Write the equation of a function that is the sum, difference, product or quotient of two or more functions, given their equations. 1.3 Determine the domain and range of a function that is the sum, difference, product or quotient of two functions. 1.4 Write a function $h(x)$ as the sum, difference, product or quotient of two or more functions. 1.5 Determine the value of the composition of functions when evaluated at a point, including: • $f(f(a))$ • $f(g(a))$ • $g(f(a))$. 1.6 Determine, given the equations of two functions $f(x)$ and $g(x)$, the equation of the composite function: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$ and explain any restrictions.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
	1.7 Sketch, given the equations of two functions $f(x)$ and $g(x)$, the graph of the composite function: • $f\left(f\left(x\right)\right)$ • $f\left(g\left(x\right)\right)$ • $g\left(f\left(x\right)\right)$. 1.8 Write a function $h(x)$ as the composition of two or more functions. 1.9 Write a function $h(x)$ by combining two or more functions through operations on, and compositions of, functions.
2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	2.1 Compare the graphs of a set of functions of the form $y - k = f(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of k. 2.2 Compare the graphs of a set of functions of the form $y = f\left(x - h\right)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of h. 2.3 Compare the graphs of a set of functions of the form $y - k = f(x - h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of h and k. 2.4 Sketch the graph of $y - k = f(x)$, $y = f\left(x - h\right)$ or $y - k = f(x - h)$ for given values of h and k, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. 2.5 Write the equation of a function whose graph is a vertical and/or horizontal translation of the graph of the function $y = f(x)$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. [C, CN, R, V]	3.1 Compare the graphs of a set of functions of the form $y=af(x)$ to the graph of $y=f(x)$, and generalize, using inductive reasoning, a rule about the effect of *a*. 3.2 Compare the graphs of a set of functions of the form $y=f(bx)$ to the graph of $y=f(x)$, and generalize, using inductive reasoning, a rule about the effect of *b*. 3.3 Compare the graphs of a set of functions of the form $y=af(bx)$ to the graph of $y=f(x)$, and generalize, using inductive reasoning, a rule about the effects of *a* and *b*. 3.4 Sketch the graph of $y=af(x)$, $y=f(bx)$ or $y=af(bx)$ for given values of *a* and *b*, given a sketch of the function $y=f(x)$, where the equation of $y=f(x)$ is not given. 3.5 Write the equation of a function, given its graph which is a vertical and/or horizontal stretch of the graph of the function $y=f(x)$.
4. Apply translations and stretches to the graphs and equations of functions. [C, CN, R, V]	4.1 Sketch the graph of the function $y-k=af(b(x-h))$ for given values of *a*, *b*, *h* and *k*, given the graph of the function $y=f(x)$, where the equation of $y=f(x)$ is not given. 4.2 Write the equation of a function, given its graph which is a translation and/or stretch of the graph of the function $y=f(x)$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
5. Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: • x-axis • y-axis • line $y = x$. [C, CN, R, V]	5.1 Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x-axis, the y-axis or the line $y = x$. 5.2 Sketch the reflection of the graph of a function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. 5.3 Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$. 5.4 Sketch the graphs of the functions $y = -f(x)$, $y = f(-x)$ and $x = -f(y)$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. 5.5 Write the equation of a function, given its graph which is a reflection of the graph of the function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$.
6. Demonstrate an understanding of inverses of relations. [C, CN, R, V]	6.1 Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation. 6.2 Explain how the transformation $(x, y) \Rightarrow (y, x)$ can be used to sketch the inverse of a relation. 6.3 Sketch the graph of the inverse relation, given the graph of a relation. 6.4 Determine if a relation and its inverse are functions. 6.5 Determine restrictions on the domain of a function in order for its inverse to be a function. 6.6 Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation. 6.7 Explain the relationship between the domains and ranges of a relation and its inverse. 6.8 Determine, algebraically or graphically, if two functions are inverses of each other.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
7. Demonstrate an understanding of logarithms. [CN, ME, R]	7.1 Explain the relationship between logarithms and exponents. 7.2 Express a logarithmic expression as an exponential expression and vice versa. 7.3 Determine, without technology, the exact value of a logarithm, such as $\log_2 8$. 7.4 Estimate the value of a logarithm, using benchmarks, and explain the reasoning; e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1.
8. Demonstrate an understanding of the product, quotient and power laws of logarithms. [C, CN, R, T]	8.1 Develop and generalize the laws for logarithms, using numeric examples and exponent laws. 8.2 Derive each law of logarithms. 8.3 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression. 8.4 Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
9. Graph and analyze exponential and logarithmic functions. [C, CN, T, V]	9.1 Sketch, with or without technology, a graph of an exponential function of the form $y = a^x$, $a > 0$. 9.2 Identify the characteristics of the graph of an exponential function of the form $y = a^x$, $a > 0$, including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote. 9.3 Sketch the graph of an exponential function by applying a set of transformations to the graph of $y = a^x$, $a > 0$, and state the characteristics of the graph. 9.4 Sketch, with or without technology, the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$. 9.5 Identify the characteristics of the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$, including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote. 9.6 Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph. 9.7 Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.
10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]	10.1 Determine the solution of an exponential equation in which the bases are powers of one another. 10.2 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies. 10.3 Determine the solution of a logarithmic equation, and verify the solution. 10.4 Explain why a value obtained in solving a logarithmic equation may be extraneous. 10.5 Solve a problem that involves exponential growth or decay. 10.6 Solve a problem that involves the application of exponential equations to loans, mortgages and investments. 10.7 Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale. 10.8 Solve a problem by modelling a situation with an exponential or a logarithmic equation.

[C] Communication	**[PS]** Problem Solving
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	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). [C, CN, ME]	11.1 Explain how long division of a polynomial expression by a binomial expression of the form $x-a, a \in I$, is related to synthetic division. 11.2 Divide a polynomial expression by a binomial expression of the form $x-a, a \in I$, using long division or synthetic division. 11.3 Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function. 11.4 Explain the relationship between the remainder when a polynomial expression is divided by $x-a, a \in I$, and the value of the polynomial expression at $x = a$ (remainder theorem). 11.5 Explain and apply the factor theorem to express a polynomial expression as a product of factors.
12. Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5). [C, CN, T, V]	12.1 Identify the polynomial functions in a set of functions, and explain the reasoning. 12.2 Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function. 12.3 Generalize rules for graphing polynomial functions of odd or even degree. 12.4 Explain the relationship between: • the zeros of a polynomial function • the roots of the corresponding polynomial equation • the *x*-intercepts of the graph of the polynomial function. 12.5 Explain how the multiplicity of a zero of a polynomial function affects the graph. 12.6 Sketch, with or without technology, the graph of a polynomial function. 12.7 Solve a problem by modelling a given situation with a polynomial function and analyzing the graph of the function.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
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	[V] Visualization

Relations and Functions (continued)	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
13. Graph and analyze radical functions (limited to functions involving one radical). [CN, R, T, V]	13.1 Sketch the graph of the function $y = \sqrt{x}$, using a table of values, and state the domain and range. 13.2 Sketch the graph of the function $y - k = a\sqrt{b(x-h)}$ by applying transformations to the graph of the function $y = \sqrt{x}$, and state the domain and range. 13.3 Sketch the graph of the function $y = \sqrt{f(x)}$, given the graph of the function $y = f(x)$, and explain the strategies used. 13.4 Compare the domain and range of the function $y = \sqrt{f(x)}$, to the domain and range of the function $y = f(x)$, and explain why the domains and ranges may differ. 13.5 Describe the relationship between the roots of a radical equation and the *x*-intercepts of the graph of the corresponding radical function. 13.6 Determine, graphically, an approximate solution of a radical equation.
14. Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, R, T, V]	14.1 Graph, with or without technology, a rational function. 14.2 Analyze the graphs of a set of rational functions to identify common characteristics. 14.3 Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value. 14.4 Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value. 14.5 Match a set of rational functions to their graphs, and explain the reasoning. 14.6 Describe the relationship between the roots of a rational equation and the *x*-intercepts of the graph of the corresponding rational function. 14.7 Determine, graphically, an approximate solution of a rational equation.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Permutations, Combinations and Binomial Theorem	**General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Apply the fundamental counting principle to solve problems. [C, PS, R, V]	1.1 Count the total number of possible choices that can be made, using graphic organizers such as lists and tree diagrams. 1.2 Explain, using examples, why the total number of possible choices is found by multiplying rather than adding the number of ways the individual choices can be made. 1.3 Solve a simple counting problem by applying the fundamental counting principle.
2. Determine the number of permutations of *n* elements taken *r* at a time to solve problems. [C, PS, R, V]	2.1 Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row. 2.2 Determine, in factorial notation, the number of permutations of *n* different elements taken *n* at a time to solve a problem. 2.3 Determine, using a variety of strategies, the number of permutations of *n* different elements taken *r* at a time to solve a problem. 2.4 Explain why *n* must be greater than or equal to *r* in the notation ${}_nP_r$. 2.5 Solve an equation that involves ${}_nP_r$ notation, such as ${}_nP_2 = 30$. 2.6 Explain, using examples, the effect on the total number of permutations when two or more elements are identical.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Permutations, Combinations and Binomial Theorem (continued)	**General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
3. Determine the number of combinations of *n* different elements taken *r* at a time to solve problems. [C, PS, R, V]	3.1 Explain, using examples, the difference between a permutation and a combination. 3.2 Determine the number of ways that a subset of *k* elements can be selected from a set of *n* different elements. 3.3 Determine the number of combinations of *n* different elements taken *r* at a time to solve a problem. 3.4 Explain why *n* must be greater than or equal to *r* in the notation ${}_nC_r$ or $\binom{n}{r}$. 3.5 Explain, using examples, why ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$. 3.6 Solve an equation that involves ${}_nC_r$ or $\binom{n}{r}$ notation, such as ${}_nC_2 = 15$ or $\binom{n}{2} = 15$.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Permutations, Combinations and Binomial Theorem (continued)	**General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
4. Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). [CN, R, V]	4.1 Explain the patterns found in the expanded form of $(x+y)^n$, $n \leq 4$ and $n \in N$, by multiplying n factors of $(x+y)$. 4.2 Explain how to determine the subsequent row in Pascal's triangle, given any row. 4.3 Relate the coefficients of the terms in the expansion of $(x+y)^n$ to the $(n+1)$ row in Pascal's triangle. 4.4 Explain, using examples, how the coefficients of the terms in the expansion of $(x+y)^n$ are determined by combinations. 4.5 Expand, using the binomial theorem, $(x+y)^n$. 4.6 Determine a specific term in the expansion of $(x+y)^n$.

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